Combined temporary teaching guide for

Seeing through arithmetic 2

Seeing through arithmetic, Practice tablet 2

Seeing through arithmetic 2, Workbook edition

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This is a Temporary Teaching Guide to accompany the first half of the hardbound *Seeing Through Arithmetic 2* (*STA 2*), the *Seeing Through Arithmetic, Practice Tablet 2,* and the *Seeing Through Arithmetic 2, Workbook Edition.*

If you are using the hardbound *Seeing Through Arithmetic 2,* you will find teaching notes for each page in the pupil's book indicated by the letters *STA* followed by page number. For example, *STA 2* page 5 indicates that the notes and directions are for page 5 of *Seeing Through Arithmetic 2.* If you are using *Practice Tablet 2* along with *Seeing Through Arithmetic 2,* directions for each page in the Practice Tablet will be labeled "Practice Tablet" followed by the Tablet page number. For example, *Practice tablet* page 6 means that the notes and directions are for page 6 of *Practice Tablet 2.*

If you are using the Workbook Edition, the directions and notes for each page will be labeled "Workbook edition" followed by the page number. For example, **Workbook edition** page 19 indicates that the notes and directions are for page 19 of *Seeing Through Arithmetic 2, Workbook Edition.*

This temporary Guide will be superseded during the school year 1964-1965 by permanent teacher's editions. The Teacher's Edition for *Seeing Through Arithmetic 2* will be a hardbound book of 672 pages. It will show, in full size and color, each page from *Seeing Through Arithmetic 2.* Complete teaching notes will be printed at the right on the page facing the page from the pupil's book. Responses for all exercises will be indicated on the reproduced pupil's page. Pages from the Tablet will be reproduced in slightly less than full size and will show all responses. The directions for the Tablet page will be printed beneath the reproduction of the pupil's page.

The Teacher's Edition for the Workbook Edition will be a paperback of 576 pages. It will show all the teaching pages in full size and color with all responses indicated on the reproduced page. Complete notes and directions for teaching the pupil's page will be printed at the right on the page facing the page from the pupil's book. The practice pages will be slightly reduced in size, and with all responses shown. Directions for using the practice page will be given beneath the reproduced pupil's page.

GEOMETRY

Geometry is a branch of mathematics that deals with such objects as points, lines, planes, and space. The study of geometry in the elementary grades is concerned with relationship among these objects. The word "objects," as used here, refers to mathematical ideas. In a formal development of geometry, the ideas of point, line, and plane are usually *undefined terms.* In organizing mathematical ideas, undefined terms are those primitive elements for which no formal definition can be given. While mathematical ideas such as point, line, and plane can be related to the physical world, it is not desirable to define them in a formal way. In the primary grades, a point may be suggested by a dot, a line by a tightly stretched wire, and a plane by the surface of a flat object, such as a chalkboard or mirror.

One way to approach the study of geometry is to utilize the idea of sets. *Set* is a mathematical term that refers to a group, or collection, of objects. If each object of one set is also contained in a second set, then the first set is a *subset* of the second set. For example, think of a set of three kittens: a yellow kitten, a black kitten, and a white kitten. The set of kittens that are not white (the yellow kitten and the black kitten) is a subset of the set of three kittens. In a similar way, space is the set of all points, and a point, a line, and a plane are subsets of space.

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The geometric ideas presented in grades 1 and 2 of the *STA Program* are concerned with *plane geometry*, which is a study of the geometry of a plane. The geometric figures considered in these books—curves, lines, segments, and polygons—are *coplanar* figures. Coplanar figures are figures that are included in the same plane. In this *Teaching Guide*, when we refer to lines we mean coplanar lines.

Geometric ideas are presented in grades 1 and 2 in such a way that they are related to the world with which children are familiar. The presentation is informal and intuitive, but it is developed in such a way that later understanding of precise mathematical ideas will be facilitated.

In grade 1, the children learned to identify an open curve and a closed curve. They also learned certain properties of these curves. In grade 2, the children extend their knowledge of open curves and the properties of open curves to include the ideas of lines, line segments, and relationships among lines. They extend their knowledge of closed curves and the properties of closed curves to include polygons.

The development of each geometric idea provides learning experiences that progress from concrete situations to abstract situations. The idea is first related to physical situations that are familiar to children. Then the idea is presented in a situation that is not related to the physical world.

STA 2 page 1

Workbook edition page 1

Reteaching the idea of curves; introduction of the line as a special curve; reteaching the idea of betweenness

The children extend their knowledge of geometry by developing an informal and intuitive notion of a line. They learn that a line is a special curve that does not bend, and they review the idea of betweenness.

On this page, the curves are represented by paths, such as a rainbow, a jet airplane trail, a river bank, and a pipe. The word "path," as used here, refers to the physical representation of a curve or a line. Since a curve or a line is an idea, it cannot be seen, but it can be represented or pictured as a path. Each picture shows part of a path that represents a curve. Have the children tell what they see in each picture.

Again direct attention to the top picture. Have the children trace the rainbow with their fingers. Ask if anyone remembers what name may be used in talking about such a path. Remind them that the rainbow represents a curve.

Direct attention to the picture of the jet airplane trail. The technical name for such a trail is "contrail." Because many children are not familiar with the term "contrail," we will refer to it as a jet trail. Have the children trace the jet trail. They should understand that the jet trail represents a curve.

Have them compare the curve represented by the rainbow and the curve represented by the jet trail. They should observe that each path is a curve, that the curve represented by the rainbow bends, and that the curve represented by the trail of the jet does not bend. Tell them that the jet trail represents a special curve called a *line*. (In geometry, a line is understood to be a "straight line." When we say "line," we will mean "straight line.") Emphasize that a line is a curve that does not bend.

Next have the children look at the picture of the bank of a river. Have them trace the bank with their fingers. Ask if this path represents a curve. Then ask if it represents a line. The children should understand that the bank of the river does represent a curve, but, since the bank tends, it does not represent a line.

Direct attention to the picture of the pipe.

Ask if the pipe represents a curve. Then ask if, as far as they can tell from the part of the pipe shown in the picture, it represents a line. The children should understand that the pipe represents a curve, and that, since the part of the pipe shown in the picture does not bend, it also represents a line.

Have the children suggest objects that represent curves that are lines and curves that are not lines. For example, they might suggest that a winding road represents a curve that is not a line and a long straight sidewalk represents a curve that is a line. Give attention to the idea that curves and lines are endless. In discussing physical models of curves and lines (for example, a sidewalk), whenever it is appropriate, stress that the model that represents a line goes "on and on."

Direct attention again to the river bank. Suggest that the children pretend they are taking a walk along the bank. Have them use their fingers to trace the bank from the bush to the boat and to name the objects that they pass. Ask: "Can you walk along the bank from the bush to the boat without passing the rocks? Can you walk along the bank from the boat to the bush without passing the rocks? Since you must pass the rocks, you can say that the rocks are between the bush and the boat. What objects do you pass as you walk from the rocks to the tree? Is the rowboat between the rocks and the tree?"

Then have the children pretend to walk along the bank from the tree to the bush and give other examples of betweenness. They should discover that the boat is between the tree and the bush and that the rocks are between the tree and the bush.

Direct attention again to the picture of the pipe. Have the children use their fingers to trace the pipe. They should see that they must always pass certain objects as they trace the pipe. They should discover that the valve is between the short cactus and the tall cactus that the short cactus is between the flowering cactus and the valve; that the valve is between the flowering cactus and the tall cactus; and that the short cactus is between the flowering cactus and the tall cactus.

STA 2 page 4

Workbook edition page 2

Identification of lines and curves; naming of points; reteaching the idea of betweenness

The children identify curves and lines represented by paths that are not related to familiar physical situations. They learn that a dot represents a point. They also name points and review the idea of betweenness.

Do not hurry the work on this page. It may be desirable to use two days for the discussion of this page. If you do, discuss pictures A, B, and C on the first day, and on the following day emphasize the same ideas during the discussion of pictures D, E, and F.

For each picture, first have the children decide if the path represents a line. They should identify the curves represented in pictures A, B, D, and F as lines because these paths do not bend. They should discover that the curves presented in pictures C and E are not lines because the paths in these pictures bend.

Refer again to picture A. Say: "Find the blue dot. This dot represents a point in the line. Now find the red dot (Wkbk, orange)*. The red dot represents another point in the line. Find the brown dot (Wkbk, gray). Does the brown dot also represent a point? Is this point in the line? Find the green dot (Wkbk, light blue). Is the green dot in the path? Does the green dot represent a point?" Ask the children if they passed through the green dot when they traced the path.

Next have the children trace the path from the blue dot to the brown dot (Wkbk, gray).

* Some color references and most page references are not the same in the Workbook Edition (Wkbk) as in the Seeing Through Arithmetic (STA 2) book. When color or page references are different from those in STA 2, they will be inserted in the text within parentheses.

Help the children discover that they must pass through the red dot (Wkbk, orange) as they trace the path from the blue dot to the brown dot (Wkbk, gray), and that they can say that the red dot (Wkbk, orange) is between the blue dot and the brown dot (Wkbk, gray). Also bring out the idea that, since the green dot (Wkbk, light blue) is not in the line, it is not sensible to ask if the green dot is between any points in the line.

Next have each child place a finger on the path between the blue dot and the red dot (Wkbk, orange). Explain that the part of the path they are touching represents a point in the line. Then have them place a finger between the red dot and the brown dot (Wkbk, gray). Help them discover that this part of the path also represents a point in the line. Repeat this activity several times, and make certain each time that the children do not place their fingers on any of the colored dots. They should discover that there are many points in the line. They should also understand that there are many points in the line that are between the blue dot and the red dot (Wkbk, orange), between the red dot and the brown dot (Wkbk, gray), and so on.

Use a similar teaching procedure to introduce the idea that there are many points that are not in the line shown in picture A. The children should understand that the green dot (Wkbk, light blue) represents one of these points and that there are many other points not represented by dots.

Direct attention to picture B. Have each child place a finger on the dot that is not on the path. Ask the children to name the letter that is printed near the dot. Explain that, in this picture, the point represented by this dot is named "point M." (Any letter of the alphabet may be used to name a point in a line.) Have the children name the other points (point J, point K, point L, point M) that are represented by dots.

Have the children place their fingers on the dot that represents point J and trace the path to point M. Then ask them to trace the path from point M to point J. They should discover that, in either case, they must pass point K and point L and that therefore point K and point L are between point J and point M (or between point M and point J). They should observe that point K is between point J and point L and that point L is between point K and point M. Be sure the children understand that there are *many* points in a line that are between any two different points in the line.

Discuss the location of point N. The children should see that point N is not in the line. Remind them that they cannot say that point N is between any of the points in the line because point N is not in the line.

Direct attention to picture C. Say "Name the points in the curve. Is point S between point R and point T? Is point T between point S and point U? Name the points that are between point R and point U. Is there a point in the curve between point R and point S?" Remind the children that there are many points in the curve, but that we talk only about the points that are named.

Direct attention to the orange dot. Have the children suggest a name for the point represented by this dot (for example, "point W"). Explain that, to name this point, they can use any letter that has not been used to name other points represented in the picture. They should observe that the orange dot represents a point that is not in the curve and that this point is not between any of the points in the curve because it is not in the curve.

Use pictures D, E, and F to reinforce the ideas previously presented. Have the children identify the labeled points, suggest names for the points that are not labeled, identify points that are between other points in each line (or curve), and identify points that are not in the line (or curve). Emphasize the idea that, for each picture, there are many

points that are not represented by dots.

Practice tablet page 1
Workbook edition page 3

Use your usual reading method to introduce the words *yes* and *no*. Explain to the children that if a path shows a line, they draw a ring around the word *yes*; if the path does not show a line, they draw a ring around the word *no*.

Direct attention to Picture A. Have the children trace the path with a pencil or crayon. Ask if the path shows a line. Ask which word, *yes* or *no*, they are going to draw a ring around. Tell them to draw a ring around this word.

Next ask them to name the points that are in the line. Tell them to put a dot between point L and point W. Remind them that this dot must be on the path. Tell them to name this dot. Be sure the children understand that they can use any letter of the alphabet, as long as it is different from the letters already used.

Follow a similar procedure for Picture B. Then have the children complete the page.

Practice tablet page 2
Workbook edition page 4

The directions for using *Practice tablet* page 1 (**Workbook edition** page 3) may be adapted to this page.

STA 2 page 5
Workbook edition page 5

Identification of curves; introduction of segments

The children have learned that a line is a special curve that does not bend. Now they are introduced to the idea of a segment.

In Euclidean geometry, a *line* is a set of points. A *segment* is a subset of a line. A segment is a set of points that is determined by two points in a line and all points between these two points. Note that we cannot define a segment simply as "a part of a line" because a point is also a part of a line. So we must define a segment as a set of points that includes two given points in a line and all the points in the line that are between the two given points.

Direct attention to the top picture. Have the children tell what they see and identify the wire as representing a line. They should think of the wire as going on and on. Suggest that they trace with their fingers the line represented by the wire. Draw attention to the poles that have colored bands near the top. Have the children start with the pole that has an orange band at the top and name each object that they pass as they move along the wire to the pole that has a blue band at the top. They should understand that, because they must pass the top of the yellow pole (Wkbk, white), they can say that the top of the yellow pole is between the tops of the other two poles. They should decide that the bird is between the top of the yellow pole (Wkbk, white) and the top of the blue pole; that the top of the yellow pole (Wkbk, white) is between the bird and the top of the orange pole; that the bird is between the top of the blue pole and the top of the orange pole; and that the top of the yellow pole (Wkbk, white) and the bird are between the top of the orange pole and the top of the blue pole.

Follow a similar procedure for each of the other pictures. Direct the discussion so that the children will understand that, since the stream bends, it represents a curve that is not a line; that, since the road does not bend, it represents a line. Have the children identify objects that are between other objects along the stream and along the road. For example, the waterfall is between the tree and the rocks; the bridge is between the

yellow sign (Wkbk, blue and orange) and the red sign (Wkbk, orange).

Refer again to the top picture. Have each child place a finger at the top of the orange pole and another finger at the top of the yellow pole (Wkbk, white). Help the children discover that the top of the orange pole, the top of the yellow pole (Wkbk, white), and all the wire between the tops of these two poles represent a particular part of a line. Tell them that a particular part of a line is called a *segment*. Explain that the top of the orange pole and the top of the yellow pole (Wkbk, white) represent the ends of this segment. Next lead the children to discover that the top of the yellow pole (Wkbk, white), the top of the blue pole, and all the wire between the tops of these two poles represent another segment in this line and that the top of the yellow pole (Wkbk, white) and the top of the blue pole represent the ends of this segment. Follow the procedures you have been using to have the children identify other segments that are represented.

Next direct attention to the picture of the stream. Select various objects that can be thought of as representing ends of a part of the curve represented. For example, the tree and the waterfall can represent the ends of a particular part of the curve. Ask the children if the stream represents a curve that is a line. After they have decided that the stream does not represent a line, explain that the term "segment" cannot be used as a name for a particular part of a curve that is not a line. Emphasize the fact that a segment is a particular part of a line. Consequently, no segments are represented in the picture of the stream.

Finally direct attention to the picture of the road. Again select objects that can be thought of as representing ends of a part of the curve represented. For example, the orange sign and the bridge can represent the ends of one particular part of the curve. Then ask: "Does the road represent a line? Together, do the orange sign, the bridge, and all the road between the orange sign and the bridge represent a segment?" Have the children identify other segments that are represented in the picture.

STA 2 page 6
Workbook edition page 6

Identification and naming of segments

The children identify curves, lines, and segments represented by paths that are not related to physical situations. They learn that two different points in a line are the endpoints of a segment. They use the names of these endpoints to name the segment.

The ends of a segment are called the *endpoints* of the segment. The letters assigned to these endpoints may be used to name the segment. Suppose, for example, you have a line that contains point A and point B. Point A, point B, and all the points in the line between point A and point B determine a segment. Since point A and point B are the endpoints of this segment, this segment can be named "segment AB" or "segment BA." Observe that the letters used to name the endpoints are also used to name the segment.

Throughout the remainder of these teaching notes, we will use the word "curve" when we refer to a curve that is not a line; we will use the word "line" when we refer to a curve that is a line.

For each picture on this page, first have the children decide if the path represents a line. Then have them name each point represented by a dot that is in the line (or curve) and each point that is not in the line (or curve). Also, have the children identify each point that is between other points in a line (or curve). For pictures E and F, ask the children to suggest letters as names for the unlabeled dots. Remind them that they may

use any letters that have not already been used in the picture under discussion.

Direct attention to picture A. Have each child place a finger on the dot labeled G and trace the path from G to H. Then have them retrace the path, this time from H to G. Ask what word can be used to name the part of the line that includes point G, point H, and all the points in the line between points G and H. Remind them that points G and H are the "ends" of the segment. Tell them that the word "endpoint" is the name of an "end" of a segment. The endpoints of the segment pictured in A are point G and point H. Discuss with the children why the word "endpoint" is a good name for the "end" of a segment.

Refer again to points G and H. Ask the children to suggest ways of naming segments. Lead them to the conclusion that, for any segment, the names of its endpoints may be used to name the segment. Explain that the segment in picture A may be called "segment GH" or "segment HG."

Next have the children look at picture B. Have each child place a finger on the dot labeled W and trace the picture of the line from W to the dot labeled X. Ask: "Are point W and point X the endpoints of segment WX (or segment XW)? Name the segment that has points X and Y as endpoints. Name the segment that has point W and point Y as endpoints."

Use a similar procedure for each of pictures C through F. For picture D, the children should understand that no segments are shown because the path does not represent a line. Emphasize that a segment is a part of *only* those curves that are lines. For each of pictures C, E, and F, the children should understand that the dots that are not in the line do not represent endpoints of segments that are represented in the picture.

Practice tablet page 3

Explain to the children that if the part of the path from one dot to the other shows a segment, they draw a ring around the word *yes*; if this part of the path does not show a segment, they draw a ring around the word *no*.

Direct attention to Picture A. Have the children trace with a pencil or crayon the path from one dot to the other dot. Ask them if this part of the path shows a segment. Ask which word, *yes* or *no*, they are going to draw a ring around. Tell them to draw a ring around this word.

Next ask the children to name the points that are in the line (or curve). Tell them to put a dot between point M and point Z. Remind them that this dot must be on the path. Tell them to name this dot. Be sure the children understand that they can use any letter of the alphabet, as long as it is different from the letters already used.

Follow a similar procedure for Picture B. Then have the children complete the page.

STA 2 page 7
Workbook edition page 7

Naming lines; naming segments; introduction of the printed words "line" and "segment"

The children have learned to identify lines and segments and to name segments. Now they learn to recognize the printed words "line" and "segment."

On this page, pictures A, B, and C make up the teaching sequence; pictures D through J provide reinforcement for the ideas that have been taught. Notice that the background color of the pictures that form the teaching sequence is different from the background color of the other pictures. Throughout the pupil's book, background color is used to indicate the pictures within a particular teaching sequence.

Use pictures A, B, and C to introduce the idea that a segment can be pictured without representing the parts of the line beyond the endpoints of the segment.

In picture A, draw attention to the picture of the line. If the word "line" is new to the children, introduce it as you would any new reading word. Then direct attention to the printed label "line 1" and to the arrow pointing to the line. Explain that "line 1" is a name for the line shown in picture A. Next have the children identify points R and S as endpoints of a segment, and have them name the segment as "segment RS" or as "segment SR."

In picture B, the children should observe that segment RS is shown again. Point out that the parts of the path beyond R and S are pictured as "dimmed off." This "dimming off" helps the children to think only of the segment that has R and S as endpoints.

In picture C, have the children observe that only segment RS is shown. However, the children should understand that points R and S and all points in segment RS that are between points R and S are part of a line, in this case line 1, even though the picture shows only the segment. Introduce the printed word "segment." Have the children read the name of the segment. Remind them that the segment may be named in two ways, "segment RS" or "segment SR."

When discussing picture D, explain the use of the wavy line. In the geometry section, the wavy line is replaced by the names of the two points that are endpoints of the segment. Have the children give the names of the endpoints, name the segment, and read the name of the segment. Then ask them to state the letters (in this case, LK) that should replace the wavy line. Discuss picture E in a similar manner. The children should discover that, in line WY, point X is between point W and point Y, that point X is included in segment WY, and that point X is an endpoint of each of two segments (segment WX and segment XY). Have them name the endpoints of segment WY, segment WX, and segment XY.

For each of pictures F through J, have the children name the endpoints of each segment pictured, name the segment that has those endpoints, and give two letters that can replace the wavy line. Do not require the children to give two names for each segment. For each of pictures H, I, and J, they should suggest three different names to replace the placeholder. For picture H, for example, the children should name segment MN (or NM), segment NO (or ON), and segment MO (or OM). In pictures H and J, have the children name the lines that are shown and labeled.

Workbook edition page 8

See the directions given for *Practice tablet* page 3.

Practice tablet page 4
Workbook edition page 9

Call attention to Picture A. Ask the children to tell what segment is named in Picture A and to name the endpoints of this segment. Then tell them to trace with a pencil or crayon the part of the path that shows segment WT.

Have the children follow a similar procedure for each of the other pictures. Be sure they understand that they are to trace only the part of the path that shows each segment named in a picture.

Practice tablet page 5
Workbook edition page 10

Explain that, on this page, each path shows a line or a segment. Direct attention to Picture A. Ask the children to name a segment that is shown in the picture. The children could name any one of the following segments: segment JL (or LJ), segment LV (or VL), and segment JV (or VJ). Tell them to write the name of this segment on the top response space. Ask them if another segment is shown. Have them name this segment and write its name on the next response space. Ask if

there is another segment shown. Have them name this segment and write its name on the last response space. Ask if all the segments that are shown in the picture have been named.

Before having the children complete the page, point out that they may not need to use all of the response spaces given in a picture.

STA 2 page 8
Workbook edition page 11

Reteaching the ideas of open curve and of closed curve; interior and exterior of a curve; and betweenness

The children identify pictured objects that represent closed curves and curves that are not closed. They review several ideas introduced in *Seeing Through Arithmetic 1*: a closed curve has an inside and an outside, but does not have betweenness for points; a curve that is not closed (an open curve) has neither an inside nor an outside; and, in pictures that contain representations of curves that are not closed, objects can be identified as being between other objects.

In *Seeing Through Arithmetic 1*, the children learned that a path represents a closed curve if they can select a starting point, trace the path with their fingers, and return to the selected starting point without retracing any part of the path. They also learned that a path represents an open curve if, when they select a starting point and trace the path with their fingers, they have to retrace the path in order to return to the selected starting point. A segment is an example of an open curve.

Direct attention to the top picture. Have the children describe what they see. Then have them select various starting points and trace the log with their fingers. They should observe that, if they start in front of the orange car (or in front of any other car) and trace the log, they cannot return to the orange car (or the selected car) without retracing the log. Lead them to discover that the log represents a segment.

Next ask: "When you trace the log, can you get from the orange car to the green car (Wkbk, light blue) without passing the blue car? Which car is between the orange car and the green car (Wkbk, light blue)? The green car is between which two cars? Which cars are between the yellow car (Wkbk, brown) and the orange car?"

Finally have the children imagine that they are getting out of one of the cars to get a drink at the water fountain. Ask: "Can you get from the yellow car (Wkbk, brown) to the drinking fountain without crossing the log? Can you get from the orange car to the drinking fountain without crossing the log?" In each case, the children should discover that they can get from any car to the drinking fountain without crossing the log. Direct the class discussion so that the children understand that, since the log represents a segment, the log has two ends and they can get to the drinking fountain by going around one of these ends.

Now direct attention to the bottom picture. Have the children describe what they see. Next have them pretend that they are going to take a walk around the pond. They may select any object along the path as a starting point, for example, the bench. Have them trace the path with their fingers and name each object they pass as they move along the path. Ask if they can return to the bench without retracing the path. Have them take several such "walks" using various objects as starting points. For each "walk," the children should discover that they can always return to their selected starting point without retracing the path.

Ask the children if the path represents a segment or a line. Then explain that the path around the pond represents a *closed curve*.

Be sure the children understand that a path represents a closed curve only if they can trace the path and return to the selected starting point without retracing any part of the path.

Next ask the children if they can walk from the tree to the flower garden without passing the bench. They should discover that they can get from the tree to the flower garden without passing the bench. They should understand that, since they can get from the tree to the flower garden without passing the bench, they cannot say that the bench is between the tree and the flower garden. Ask if the flower garden is between the bench and the tree; if the tree is between the flower garden and the bench. Be sure the children understand that they cannot identify an object as being between other objects along a path that represents a closed curve.

Suggest that the ducks are going for a swim in the pond. Ask if the ducks can get to the pond without crossing the path. Explain that you can think of the pond as being *inside* the walk and that the ducks are *outside* the walk. Mathematically, a closed curve has an interior rather than an inside, and an exterior rather than an outside. However the words "inside" and "outside" may be more meaningful to children at this level. Then ask: "Is the tree inside or outside the closed curve represented by the path? Is the flower garden inside or outside the closed curve represented by the path? When the ducks are swimming, will they be inside or outside the closed curve?"

Finally, have the children compare the two pictures. In comparing the pictures, bring out the following ideas:

1 The log represents a segment (it has two ends and does not bend).
2 A segment is not a closed curve (as the children traced the log, they had to retrace part of the log to return to the starting point).
3 There is betweenness for a segment (to get from the yellow car (Wkbk, brown) to the blue car, they had to pass the green car (Wkbk, light blue), and so on).
4 There is no inside or outside for a segment (they could get from any car to the water fountain without crossing the log).
5 The walk represents a closed curve (they did not have to retrace the walk to return to the starting point).
6 There is no betweenness for a closed curve (to get from the tree to the flower bed, they did not have to pass the bench).
7 A closed curve has an inside and an outside (the ducks had to cross the path to get into the pond).

STA 2 page 9
Workbook edition page 12

Identification of open curves and of closed curves; interior and exterior of curves; betweenness

The children decide for each path whether it represents a closed curve or a curve that is not closed. They also decide if a point is between other points in the curve and if points are inside or outside the curve.

The paths on this page represent curves and parts of curves that are not related to physical situations. The paths in pictures A, B, D, F, and H represent closed curves. Those in pictures C, E, G, and I represent curves that are not closed.

The pictures may be discussed in any order. For each picture, first have the children decide whether the path represents a closed curve or a curve that is not closed. For pictures that show curves that are not closed, the children should identify each path that represents a segment (C, G, and I).

Next have the children decide whether a point is inside or outside the curve. For

each of pictures A, B, D, F, and H, have them discover which point or points are inside the curve and which point or points are outside the curve. Use questions to help them recall that many other points that are not represented by dots are also inside or outside the curves. When discussing pictures C, E, G, and I, the children should understand that a curve that is not closed has no inside or outside.

Then have the children decide if a point is between other points in the curve. For each of pictures C, E, G, and I, have them name three points and identify the point that is between the other two selected points. For picture E, the children may identify the points by the colors of the dots representing the points, or they may supply labels. They should understand that many other points not represented by dots are also between each pair of points in the curve. When discussing pictures A, B, D, F, and H, the children should understand that there is no betweenness for points in curves that are closed.

Practice tablet page 6
Workbook edition page 13

Call attention to Picture A. Have the children place their pencils or crayons on one of the dots and trace the path. Ask if the path shows a closed curve or an open curve. Ask if they can show a point that is between point V and point N. The children should understand that, for a closed curve, there are no points between two other points. Then ask if a closed curve has an inside. Tell the children to put a dot to show a point that is inside the closed curve.

For Picture B, have the children place their pencils on one of the dots and trace the path. Ask if this path shows a closed curve or an open curve. Ask if an open curve has an inside. The children should understand that an open curve does not have an inside or an outside. Then ask if they can show a point that is between point Z and point Y. Tell them to put a dot on the path to show a point that is between point Z and point Y.

Have the children complete the page. For each picture, they first trace with pencils or crayons the path. Then, if a closed curve is shown, they put a dot to show a point that is inside the curve. If an open curve is shown, they put a dot on the path to show a point that is between the two points given.

Practice tablet page 7
Workbook edition page 14

Direct attention to Picture A. Have the children place their pencils or crayons on one of the dots and trace the path. Ask if this path shows an open curve or a closed curve. Ask if an open curve has an outside. Then ask if they can show a point that is between point M and point Z. Tell them to put a dot on the path to show a point that is between point M and point Z.

For Picture B, have the children place their pencils on one of the dots and trace the path. Ask if the path shows an open curve or a closed curve. Ask if they can show a point that is between point Q and point R. Then ask if a closed curve has an outside. Tell them to put a dot to show a point that is outside the closed curve.

Have the children complete the page. For each picture, they first trace with pencils or crayons the path. Then, if a closed curve is shown, they put a dot to show a point that is outside the curve. If an open curve is shown, they put a dot on the path to show a point that is between the two points given.

Introduction of intersecting lines and parallel lines

The children have learned that a line is a special kind of curve that does not bend and that a segment is a particular part of a line. Now they study intersecting lines and parallel lines.

In geometry, if a point, for example, point M, is contained in line 1 and also in line 2, we say that the two lines *intersect*. Point M, in this case, is the *point of intersection*. If two lines intersect, their intersection contains exactly one point.

If two lines do not intersect and are in the same plane, the lines are *parallel*. For example, if no point in line 3 is contained in line 4, line 3 and line 4 are parallel lines. It is important to remember that lines 3 and 4 are subsets of the same plane (that is, they are coplanar lines).

When you discuss the pictures on this page, use the terms "intersecting lines" and "parallel lines."

Direct attention to the top picture. Have the children describe what they see. They should observe that the cement walks represent lines. Have them trace each walk with their fingers and name the objects they pass. After discussing the objects in the picture, ask: "If you walk from the fountain to the flower garden, do you pass the orange cement block? If you walk from the lamp to the green cement block (Wkbk, gray), do you pass the orange block? Is the blue block in the same walk as the green block (Wkbk, gray)? Trace the walks from the blue block to the green block (Wkbk, gray). Do you have to pass the orange block? Is the orange block a part of the walk that has the blue block? Is the orange block a part of the walk that has the green block (Wkbk, gray)? Is the orange block a part of both walks?"

Have the children take several other "walks" (from the bench to the flower bed, from the drinking fountain to the lamp, for example), to reinforce the idea that the orange block is the only object that is contained in both walks. Then explain that, since the two cement walks contain the same orange block, the walks *intersect*. Tell them that the walks represent *intersecting lines*.

Next direct attention to the middle picture. Have the children describe what they see. They should observe that the walks represent lines. Again have them trace each walk with their fingers and name the objects they pass. Then ask: "Is the orange block of cement a part of both walks? Is the blue cement block in both walks? Is there any way to get from one walk to the other walk without going across the grass? Is any part of the walk having the orange block a part of the walk having the blue block?" Help the children discover that no block of cement is a part of both walks. In response to further questions about the walks, the children should say that these two walks do not represent intersecting lines. Suggest that another name is needed for lines that do not intersect. Tell them that *parallel lines* is the name for lines that do not intersect.

Refer to the bottom picture. In this picture some cement walks may be used as representations of intersecting lines and others as representations of parallel lines. You can identify a walk by referring to the colors of the two blocks in the walk or by the objects shown along the walk. Have the children identify colored blocks that belong to more than one walk. Have them also select walks that represent intersecting lines and walks that represent parallel lines.

STA 2 page 11
Workbook edition page 16

Identification of intersecting lines, parallel lines, and points of intersection

The children identify paths that represent intersecting lines and parallel lines. They learn about points of intersection and points that are not points of intersection. They also identify parts of paths representing segments that are parts of intersecting lines and of parallel lines.

For picture A, have the children identify line 1 and line 2. Explain that the labeling ("line 1" and "line 2") is simply a matter of choice and that any numerals (and any order) might have been used.

Ask: "What points in line 1 are named? What points in line 2 are named? What point is in both line 1 and line 2? Are line 1 and line 2 intersecting lines or parallel lines? Since line 1 and line 2 are intersecting lines, and since point J is in both line 1 and line 2, you can call point J the *point of intersection*. Is point K a point of intersection? Is point L a point of intersection?" Be sure the children understand that, if two lines intersect, only one point is the point of intersection.

For picture B, have the children read the names of the lines and give the points named in each line. They should discover that no point in line 3 is also in line 4 (and that no point in line 4 is also in line 3). Ask them what these lines are called. Direct attention again to line 3. Have the children observe that point R and point S are endpoints of a segment. They should understand that point R, point S, and the points that are between these points may be identified as segment RS (or segment SR). Then they should identify segment TU (or segment UT) in line 4.

For each of pictures C through I, have the children identify the paths as representing either intersecting lines or parallel lines; name each point of intersection; name the points that are not points of intersection; identify and name segments that are pictured; and name the points that are endpoints of more than one segment. Notice that, in pictures C and G, both intersecting lines and parallel lines are shown. For pictures C, D, F, G, and H, be sure the children can identify the intersecting lines. For pictures F and H, they should understand that the blue and red (Wkbk, orange) lines are not parallel, even though the point of intersection is not shown in the pictures.

Practice tablet page 8
Workbook edition page 17

Explain to the children that if a picture shows lines that intersect, they draw a ring around the word *yes*; if the picture shows lines that do not intersect, they draw a ring around the word *no*.

Direct attention to Picture A. Have the children trace with a pencil or crayon each path. You might want to provide each child with a straightedge to help him trace the paths. A ruler, tongue depressor, or strip of cardboard could be used as a straightedge. Show the children how to place their straightedge on the paper.

After the children have traced each path, ask if the picture shows lines that intersect. Ask which word, *yes* or *no*, they are going to draw a ring around. Tell them to draw a ring around this word.

Next ask the children to name a point of intersection that is shown in the picture. The children could name either point M or point S. Tell them to write the letter that names this point on the response space. Ask if another point of intersection is shown. Tell them to write the letter that names this point on the same response space. Then ask if all the points of intersection that are shown in the picture have been named.

For Picture H, have the children trace each path. Ask if the picture shows lines that intersect. Ask which word, *yes* or *no*, they are going to draw a ring around. Tell them to draw a ring around this word. Ask if any points of intersection are shown in the picture.

Have the children complete the page independently.

Practice tablet page 7
Workbook edition page 12

Explain to the children that if a picture shows lines that intersect, they draw a ring around the word *yes*; if the picture shows lines that do not intersect, they draw a ring around the word *no*.

Direct attention to Picture A. Have the children trace with a pencil or crayon each path. Ask if the picture shows lines that intersect. Ask which word, *yes* or *no*, they are going to draw a ring around. Tell them to draw a ring around this word.

Next explain to the children that they are going to show and name each point of intersection. Ask how many points of intersection they should show for Picture A. Tell them to put a dot to show each of the two points of intersection. Ask them to tell how they name a point. Have them name each point of intersection.

Have them complete the page independently.

STA 2 page 10
Workbook edition page 10

Identification of points, lines, segments, points of intersection, and intersecting lines

The children have learned that there are many points in a line. Now they learn that two given points determine a line and that two given points are contained in only one line.

The lesson on this page involves the following geometric idea that is common to, or characteristic of, any two points: "For any two points, there is exactly one line that contains both of these points." (When we say that a line "contains" a point, we mean that the point is in the line.)

Since two points are contained in only one line, the two points *determine* the line. For example, in picture B, the points R and T determine line 1; in picture C, points R and S determine line 2; and so on.

Points that are contained in the same line are called *collinear points*. In picture D, points S and T are collinear points. Of course, a given line contains many points, all of which are collinear with respect to the given line. But because any two points are contained in exactly one line, any two given points are collinear (that is, any two points are contained in a line).

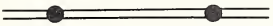
Think about three given points that are not collinear (in other words, think of three points that are not contained in the same line). These three points, taken two at a time, can be arranged as pairs in three different ways. Thus, the three points, R, S, and T, in picture A, may be paired in the following ways: points R and T; points R and S; and points S and T. Each pair of points determines exactly one line. Therefore, the three noncollinear (not in the same line) points determine exactly three lines. This idea is developed in the teaching sequence that involves pictures A through E.

Direct attention to pictures A through E. Explain that the dots in picture A represent the same points as those represented in pictures B through D. Refer to picture A and have the children name the points that are represented by dots.

Next direct attention to Picture P. Have the children identify the red path (Wkbk, orange) as representing a line and read the name of the line ("line 1"). Then ask them to

name the points that are in line 1. Explain that there is only *one* line that can contain points R and T.

Some children may argue that it is possible to draw pictures of more than one line through two pictured points as, for example, in this drawing:



It may be necessary to discuss the idea of "points" in greater detail. If so, emphasize that the colored dots used in the pictures represent points but are not points. Use illustrations such as the tip of a sharp pencil, the tip of a needle, and so on, to help children gain an understanding of what is meant by a point.

Now have the children name the point that is not in line 1, and ask if a line *could* contain points R, S, and T. They should understand that there is no line that could contain the three points represented in the picture.

Use a similar procedure for pictures C and D. Emphasize that any two of the points are contained in only one line and that no line can contain all three points that are represented in each picture.

Use picture E to emphasize the ideas that have been presented. Be sure that E is viewed as a combination of pictures A, B, C, and D. Have the children name each of the lines represented and identify the points in each line. They should observe that each point represented by a dot (for example, point R) is a point of intersection of two lines (in this case, line 1 and line 2), and they should decide that the lines are intersecting lines.

For picture F, have the children name each point represented. Ask the children if they can imagine a line that contains point M and point N. Then ask if more than one line can contain both points. The children should decide that it is not possible to have two lines that contain both point M and point N, and they should understand that point M and point N determine only one line.

Have the children identify point W, point X, and point Y in picture G. They should observe that point W and point Y are in line 4 and that point X is not in the line. They should decide that there is only one line that contains both point W and point Y and that the line that contains these two points cannot also contain point X. Ask if a line can contain point X and point Y; if a line can contain point W and point X. Direct the discussion with questions in such a way that the children conclude that there is a line that contains points X and Y and a line that contains W and X.

Follow a similar procedure for pictures H and I.

Practice tablet page 10
Workbook edition page 20

Provide each child with a straightedge. Call attention to Picture A. Tell the children that this picture has been partly done for them and will show them what they are to do for each of the other pictures on the page.

Have the children name the points that are labeled in Picture A. Ask them to tell how many lines are determined by the two points, point Z and point Q. Ask if one of the paths shows this line. Tell them to use their straightedges to draw the line determined by point Z and point Q. They should use the path that contains points Z and point Q to help them draw the line. Follow a similar procedure for point Z and point O and for point Q and point O.

If necessary, discuss Picture B with the children. Then have them complete the page. For each picture, they draw the line that is determined by each pair of points.

STA 2 page 13
Workbook edition page 21

Introduction of polygons as special closed curves

The children have learned to identify intersecting lines and to name points of intersection. They have named segments that are parts of intersecting lines. Now they learn to identify polygons as special closed curves that are made up of segments.

Polygons are closed curves composed of segments. The segments that make up a polygon are called *sides* of the polygon. A *vertex* of a polygon is a point that is a common endpoint of two of its sides. The geometric development of the idea of a polygon in grade 2 provides readiness for meaningful identification of special closed curves in grade 3.

Pictures A, B, and C are a teaching sequence. Pictures D, E, and F form a second teaching sequence. For each sequence, explain that a representation of the same geometric figure is shown in each picture.

For picture A, first have the children identify line 1, line 2, and line 3 by naming the points that determine each of these lines. Next have them name the lines that intersect and the points of intersection.

Direct attention to picture B. Observe that, for each line, the paths are dimmed off beyond the representations of the endpoints of the segments. This helps to focus attention on the segments shown in the picture. Have the children identify the endpoints of each segment and name the segments that are represented. Ask them to name the segments that have point R as an endpoint; that have point T as an endpoint; and that have point S as an endpoint.

Have the children describe what they see in picture C. They should observe that only the three segments RS, ST, and RT are pictured. Have the children place their fingers on the dot labeled S and trace the curve represented. They should discover that they can trace the path to the dot labeled R, then to the dot labeled T, and then back to the dot labeled S without going over any part of the path twice. Ask what kind of curve is pictured. The children should identify the figure that is pictured as a closed curve. Explain that this closed curve is made up of segments. Have the children name the segments. Introduce the term *polygon* as the name of a closed curve that is made up of segments.

Follow a similar procedure for the teaching sequence that consists of pictures D, E, and F. Notice that, in this sequence, four lines (and four segments) are represented. The children should understand that the figure in picture F represents a closed curve, is made up of segments, and is also called a polygon.

STA 2 page 14

Recognition of closed curves that are polygons; extension of the ideas of closed curves and of open curves, the interior and exterior of curves, and betweenness for curves

The children have learned that polygons are closed curves that are made up of segments. Now they review betweenness for curves and the interior and exterior of curves. They compare polygons, closed curves that are not polygons, and curves that are not closed. They discover that some of these geometric figures have like characteristics and that others have unlike characteristics.

Notice that the figures shown in picture A on page 14 and picture C on page 13 are alike; the figures shown in picture B, page 14, and picture F, page 13, are also alike. Point out this similarity *after* you have completed page 14.

For each picture, first have the children decide whether a closed curve or an open curve is represented. They should identify the

curves represented in pictures A, B, and F as polygons; the curve represented in picture D as a closed curve that is not a polygon; the curve represented in picture E as a curve that is not closed; and the curve represented in picture C as a segment. Next have them observe that, for each picture, some points represented are in the curve and some points represented are not in the curve. Then have them decide whether, for a given curve, a point in the curve is between other points in the curve. For each of the curves represented in pictures A, B, D, and F, they should decide that they cannot say that a given point in the curve is between other points in the curve. For pictures C and E, they should identify various given points as being between other specified points in each curve.

Then ask the children to look at various dots that represent points that are not in the curves and decide, for each such point, whether it is inside or outside the curve. For pictures C and E, they should observe that curves that are not closed do not have an inside or an outside. It is important to remember that the children also have learned to identify a figure such as that shown in picture C as a segment and should understand that no segment has an inside or an outside. For each of pictures A, B, D, and F, have them name each point that is inside the curve and each point that is outside the curve.

Have the children compare the curves that are represented. Ask how the curves are alike; then ask how the curves are different. They might say, for example, that the curve represented in picture A and the curve represented in picture D are alike because both curves are closed curves; that both curves have an inside and an outside; and that there is no betweenness for either curve. They might note differences by saying that the curve represented in picture A is made of segments, while there are no segments in the curve shown in picture D. Similarly, have the children make comparisons between the various polygons; between a polygon and a segment; and so on.

Practice tablet page 11
Workbook edition page 22

Explain to the children that if a path shows a polygon, they draw a ring around the word *yes*; if the path does not show a polygon, they draw a ring around the word *no*.

Call attention to Picture A. Have the children place their pencils or crayons on one of the dots and trace the path. Ask if the path shows a polygon. Ask which word they are going to draw a ring around. Tell them to draw a ring around this word. Next ask if a polygon is a closed curve. Ask if a closed curve has an inside. Tell the children to use a dot to show a point that is inside the curve.

Call attention to Picture B. Have the children trace the path. Ask if the path shows a polygon. Ask which word they are going to draw a ring around. Tell them to draw a ring around this word. Next ask if a closed curve has an inside. Tell the children to use a dot to show a point that is inside the curve.

Have the children complete the page. For each picture, they trace the path and draw a ring around the appropriate word. Then, if a closed curve is shown, they use a dot to show a point that is inside the curve.

STA 2 page 15
Workbook edition page 23

Identification of vertices and sides of polygons

The children have learned that a polygon is a closed curve that is made up of segments. Now they learn to identify the sides and vertices of a polygon.

Direct attention to picture A. The children should identify the curve represented as a closed curve; then they should decide that the closed curve is a polygon. Ask them to name the segments that make up the polygon. Introduce *sides* as the name for the segments that make up a polygon. Ask them to tell how many sides the polygon has and to name each side. The children should state that the polygon in A has four sides and that the names of these sides are segment WX, segment XY, segment YZ, and segment ZW. Using questions, help them discover that each labeled dot in these pictures represents the endpoint of two sides (segments). Introduce the term *vertex* as the name for each point that is an endpoint of two sides of a polygon. Have them identify each of the points, W, X, Y, and Z, as a vertex of the polygon.

Discuss each polygon represented in pictures B through H in a similar manner. The children should name the sides and the vertices of each polygon. The more able pupils could be given an opportunity to draw representations of polygons. Some of these children may discover the generalization that the number of sides and the number of vertices are the same.

At this time, the terms *three-sided polygon, four-sided polygon,* and so on could be introduced. This terminology is acceptable until such time as the names of the polygons (triangle, square, and so on) are introduced in later study.

CARDINAL IDEA OF NUMBER

The question "How many?" calls for a quantitative answer, for example, three, six, or eight. Usually the nature of the objects being counted is indicated by such replies as "three dogs," "six boys," or "eight chairs." These answers involve the cardinal idea of number. This concept is concerned with the numerousness of a set of objects and is not dependent upon the idea of counting. The cardinal idea of number grows out of the ideas of one-to-one correspondence and equivalence of sets.

A one-to-one correspondence is a special matching, or pairing, between two sets in which each member of a given set of objects is matched with one member of a second set, and each member of the second set is matched with one member of the given set. When the members of sets can be placed in one-to-one correspondence, the sets have the same number of members and are called equivalent sets.

The basic idea of one-to-one correspondence for sets leads to the development of the concept of equivalence for sets and the idea of number as a common property, or characteristic, of equivalent sets. For example, think about the set whose members are four girls (Jane, Patty, Linda, and Sally) and all sets equivalent to this set. These sets have a common property, namely, four. The physical nature of the objects in each set may be disregarded, but the concept of numbers remains.

STA 2 pages 16 and 17
Workbook edition pages 24 and 25

Recognition of the numerousness of sets associated with the numbers 0 through 10; identification of the numerals for each number [reteaching]

The children recognize the numerousness of a set of objects and identify the printed word and symbol used to name the number associated with the set.

Although it is important to keep in mind that a number is a mathematical idea, it is not necessary to force this concept upon the child in the second grade. Words and symbols, such as "nine," "8," "three," and so on, are names for numbers. Such names are called

numerals. In class discussion, try to maintain the language distinction between the idea of the number associated with a set and the names for this number. If you do this, the children will learn to use the words "number" and "numeral" correctly, without your having made an issue of this distinction.

The pictures on these pages may be discussed in any order. We suggest that you refer to the objects in each picture as a *set* of objects. The word "set" is synonymous with "group" or "collection." In talking with the children, it is better to use one term, preferably "set," consistently. The objects belonging to a set are sometimes called *members* of the set. This term is used in these teaching notes, but it need not be used with the children at the present time.

For each picture, the children should first name the number that is associated with the set of objects. Use the lists above the pictures to introduce the printed words and symbols for the numbers if the children are not familiar with them. Then use the lists for practice in recognizing the words and symbols. When you think the children are thoroughly familiar with the words and symbols, print them on the chalkboard.

After each picture has been discussed, have the children again read aloud the printed words and symbols in the lists at the top of the pages.

You may wish to provide each child with a set of cards that they can use as responses. On eleven cards, write 0 through 10, one symbol to each card. On another set of eleven cards, write "zero" through "ten," one word to each card. If you prefer, you may write the word on one side of the card and the symbol on the other side. The children can then hold up these cards as responses to questions concerning the numerousness of the pictured sets.

Practice tablet page 12
Workbook edition page 26

Have the children observe that each row begins with one of the numerals 0 through 10. Tell them that first they are to trace with pencil or crayon the numeral on each of the first two response spaces of a row. Then they are to write this numeral on each of the remaining response spaces in that row.

Have the children do the exercises on this page independently.

Practice tablet page 13
Workbook edition page 27

Direct attention to Picture A. Ask how many leaves are shown. Tell the children to write, on the response space, the numeral for this number.

Have the children complete the page. For each picture, they decide how many objects are shown and write, on the response space, the numeral for this number.

Practice tablet page 14
Workbook edition page 28

Direct attention to Picture A. Ask how many balls are shown. Tell the children to write, on the response space, the numeral for this number.

Have the children complete the page. For each picture, they decide how many objects are shown and write, on the response space, the numeral for this number.

STA 2 page 18
Workbook edition page 29

Ideas of "greater than," "less than," and "equal to" [reteaching]

The children review the ideas of "greater than," "less than," and "equal to." They

read the printed words that express these ideas and use placeholders in mathematical sentences.

In mathematics, the ideas of "greater than" and "less than" are relations of *inequality*. It is important that teachers be familiar with certain language distinctions associated with the ideas of inequality. These distinctions may be expressed as follows: "There are *more* (or *fewer*) objects in one set than in another set" and "One number *is greater than* (or *is less than*) another number." "More" and "fewer" are used when talking about objects. "Greater " and "less" are used when comparing numbers. The teacher should maintain these language distinctions in class discussion, but the children should not be required to make the distinctions.

Sets that can be placed in one-to-one correspondence have the *same number* of members and are *equivalent* sets. The term "equivalence" is used when a one-to-one correspondence exists between the members of two sets. The term "equality" is used to refer to the number associated with equivalent sets. For example, look at picture D on this page. A one-to-one correspondence exists between the members of these two sets. The set of horns is *equivalent* to the set of drums. Since the sets are equivalent, the same number is associated with each set. Therefore, the number of horns *is equal to* the number of drums, or, in other words, 6 is equal to 6. Mathematically, "is equal to" means "is the same as." It is not correct to say that the set of horns is equal to the set of drums.

To distinguish consistently between equivalence and equality requires considerable sophistication. Keep the distinction in mind, but do not require the children to use the words "equivalent," "equivalence," and "equality" or ask them to make these distinctions.

A *placeholder* in a mathematical sentence is a symbol that holds a place for the name of a number or an idea. Any symbol can be used as a placeholder. For example, ?, ■, *x*, *n*, and so on, may be used in mathematical sentences in the way that ■ and *n* are used in ■ × 2 = 6 and 8 + *n* = 10. In *Seeing Through Arithmetic 1*, the children were introduced to the square screen (■) and the round screen (●) as placeholders. Two placeholders were used because in some instances different numerals were needed as replacements in the same sentence. It is preferable to refer to ■ as a "square screen" and to ● as a "round screen." The terminology "the square" and "the circle" should be avoided because this phraseology may interfere with the clarity of certain geometric ideas concerning the square and the circle that are developed in grade 3.

You may wonder why certain symbols are introduced in *Book 2*, while others are omitted. The authors gave careful consideration to the selection of symbols introduced in the pupil's book. They believe that before children are introduced to a mathematical symbol, they should understand the basic mathematical idea represented by the symbol. The use of a symbol does not necessarily help children understand the basic idea involved and may actually create confusion and hinder understanding if used too soon. Also, the authors feel strongly that extensive practice with the symbols should follow their introduction. Because some mathematical symbols, such as ">" for "is greater than" and "<" for "is less than," have limited use at the elementary level, it was decided that there would be little or no practical value in introducing these symbols in grade 2.

Have the children compare the two sets of objects in picture A to see if the sets are equivalent. Have them trace each line with their fingers, if necessary. ("Line" as used here does not relate to the geometric notion of a line.) Help guide them to the conclusion that there are more balls than gloves and that there are fewer gloves than balls.

Then have them name the number associated with each set. By directing class discussion with appropriate questions, help the children discover that, since there are more balls (5) than gloves (3), 5 is greater than 3, and that, since there are fewer gloves than balls, 3 is less than 5.

If the children have difficulty in understanding these ideas, let them compare various sets of objects (blackboard erasers, paper cups, corks, etc.) Stress the ideas involving inequality by using sets that do not contain the same number of objects. Emphasize the idea that one set contains more (or fewer) objects than the other set. At no point in this lesson should the children be asked *how many more* (or *how many fewer*) objects there are in one set than in the other or *how much greater* (or *how much less*) one number is than another.

If the printed phrases "is greater than" and "is less than" are new to the children, introduce the phrases as you would any new reading words. Then have the children read the sentences below picture A.

Follow a similar procedure for each of the other pictures on the page. When discussing picture C, help the children discover that the number of tables is the same as the number of chairs and have them verbalize the comparison between the two sets. Encourage them to say that the number of tables (2) is equal to the number of chairs (2). Use your established reading method to introduce the printed words "is equal to." Then have them read the sentence below the picture.

Notice that, in pictures E and F, the lines pairing the members of one set with the members of the other set have been omitted. When discussing picture E, explain that the "square screen" (▩) holds a place for a numeral and the "round screen" (◍) holds a place for another numeral. Using appropriate questions, lead the discussion so that the children will understand that the first sentence below picture E is read "*What number* is less than *what other number?*" Then have the children read both sentences by replacing the placeholders with numerals that correctly complete the sentences.

You may wish to provide each child with a set of eight cards, with one of the following symbols or words printed on each card: ▩, ◍, is, than, to, greater, less, equal. Arrange situations that enable the children to compare sets of objects (for example, 4 red blocks and 1 yellow block). They can use their cards to build sentences such as "▩ is greater than ◍." Then, using the cards on which the numerals are printed, they should replace the ▩ and the ◍ with appropriate numerals.

Workbook edition page 30

Direct attention to Picture A. Ask how many cars are shown. Ask how many trucks are shown. Have the children notice the sentences in the picture. Ask if the top sentence can be used with the picture. Have a child read the top sentence aloud, supplying the missing numerals. Tell the children to write the correct numeral on each response space in this sentence. Then ask if the bottom sentence can be used with the picture. Have a child read the bottom sentence aloud, supplying the missing numerals. Tell the children to write the correct numeral on each response space in this sentence.

Direct attention to Picture B. Ask how many hats and how many purses are shown. Ask if the top sentence can be used with this picture. Tell the children that, since this sentence cannot be used with the picture, they should draw a line through the top sentence. Then ask if the bottom sentence can be used with this picture. Have a child read the sentence aloud, supplying the missing numerals. Tell the children to write the correct numeral on each response space in this sentence.

Have the children complete the page. For each picture they decide how many objects are shown in each part of the picture. Then they complete each of the sentences that can be used with the picture. Remind them to draw a line through a sentence that cannot be used with a picture.

Workbook edition page 31

Tell the children that, for each picture, they first decide how many objects are shown in each part of the picture. Then they complete each of the sentences that can be used with the picture. Remind them that, if a sentence cannot be used with a picture, they are to draw a line through this sentence.

Workbook edition page 32

Tell the children that, for each picture, they first decide how many objects are shown in each part of the picture. Then they complete each of the sentences that can be used with the picture. Remind them that, if a sentence cannot be used with a picture, they are to draw a line through this sentence.

STA 2 page 13

Workbook edition page 33

Ideas of "greater than," "less than," and "equal to" (reteaching)

The children continue to review the ideas of "greater than," "less than," and "equal to." They study the use of a wavy line (~~~) in a mathematical sentence.

Have the children compare the two sets of objects as you discuss picture A. Using appropriate questions, lead them to conclude that there are more whistles than hats and that there are fewer hats than whistles. Then have them name the number associated with each set. Direct your questioning to aid them in observing that, since there are more whistles (9) than hats (6), 9 is greater than 6, and that since there are fewer hats than whistles, 6 is less than 9. Next have them read the sentences and replace the placeholders with numerals that correctly complete the sentences.

Follow a similar procedure for each of the other pictures. Introduce the use of a wavy line (~~~) as a placeholder for a word, a phrase, or a group of words. Explain that on this page the wavy line is to be replaced by one of these groups of words: "is greater than," "is less than," or "is equal to." Then have the children read the sentences below each picture and supply the correct replacements for the placeholders.

You may wish to provide each child with an additional display card for the wavy line. If so, have the children build (or draw) sets of objects to illustrate comparisons between sets of objects and use their cards to build sentences that correspond to these situations; for example, "■ is less than ●" and "3~~~6." Then they should replace the cards labeled with "■," "●," and "~~~" by cards labeled with numerals or groups of words.

At no point in this lesson is it wise to ask *how many more* or *how many fewer* objects there are in one set than in another, or *how much greater* or *how much less* one number is than another.

Practice tablet page 15

See the directions given for **Workbook edition** page 30.

Practice tablet page 16

See the directions given for **Workbook edition** page 31.

Practice tablet page 17

See the directions given for **Workbook edition** page 32.

STA 2 pages 20 and 21
Workbook edition pages 34 and 35

Ideas of order and betweenness for numbers [reteaching]

The children review the ideas of "one less than" and "one greater than" and use these ideas to order the numbers from 0 through 10. They also use the ideas of "greater than" and "less than" to decide what number is between two other numbers.

In *Seeing Through Arithmetic 1,* the children learned to order the numbers used in counting. These numbers belong to the set of *counting numbers* (the set whose members are 1, 2, 3, 4, and so on). Now the children extend their knowledge of order to include the *natural numbers* (the set whose members are 0, 1, 2, 3, and so on). Although the teacher should be aware of the difference between the set of counting numbers and the set of natural numbers (the set of natural numbers includes zero), it is not essential that the children make this distinction. The set of counting numbers is a subset of the set of natural numbers, but the study of number in grades 1 and 2 is primarily concerned with the set of counting numbers. The number 0 is the least number of the set of natural numbers; there is no greatest number because the natural numbers continue without end.

The members of the set of natural numbers can be ordered by using the idea of "one greater than." Since 0 is the least number in the set of natural numbers and 1 is one greater than 0, 1 is the natural number that comes after 0. Since 2 is one greater than 1, 2 is the number that comes after 1. By continuing to use the idea of "one greater than," you can show that 3 comes after 2, that 4 comes after 3, and so on. The idea of order can also be developed in a similar way by using the idea of "one less than." However, at this level, it seems advisable to emphasize the idea of "one greater than," rather than the idea of "one less than," to develop the idea of order.

The children have studied the idea of betweenness for curves. Now they review betweenness for numbers. Since the natural numbers are ordered, each member of the set (except 0) can be located between two other members; for example, 3 is between 2 and 4 because 3 is one greater than 2 and 3 is one less than 4. The ideas of order and betweenness for numbers are important because they lead to meaningful, rather than rote, counting.

Direct attention to the pictures at the top of these two pages. First have the children name the number associated with each set. They should understand that each set has one member more than the set it follows (the set on the left), and that each number is one greater than the number it follows (the number on the left). Have them trace the lines that indicate the pairings, if necessary.

Then direct attention to the numerals. Explain that, since the numbers have been ordered by using the idea of "one greater than," the numerals are written in order. Finally, ask the children what number is between 6 and 8 (or some other pair of numbers). Ask them to explain how they know that 7 is between 6 and 8. The children should understand that 7 is between 6 and 8 because 7 is one greater than 6 and one less than 8 (or 8 is one greater than 7).

The exercises below the pictures are confined to the ideas of "one greater than," "one less than," and betweenness for numbers. If any of the words in these phrases are new to the children, introduce them as you would any new reading words. Use the exercises below the pictures to strengthen understanding of

the ideas involved. For each exercise, have the children read the sentence silently and decide what numeral correctly completes the sentence. Then ask one child to read the sentence orally and name the correct replacement for the placeholder. Remind them that the sentence "9 is one greater than ■" is read "Nine is one greater than *what number.*"

If the children need help with the basic ideas involved, encourage them to use the pictures and numerals at the top of these pages. Their understanding will be strengthened by building sets using concrete objects. A flannel board could be used to good advantage in connection with the activities for these pages.

Practice tablet page 18
Workbook edition page 36

Notice that this page consists of two blocks of exercises. The exercises in the first block are labeled with blue letters. The exercises in the second block are labeled with white letters.

Call attention to the exercise labeled with the blue letter A. Ask the children to read the sentence silently and to decide what numeral they should write on the response space. Then have a child read the sentence aloud, supplying the missing numeral. Tell the children to write the correct numeral on the response space. If necessary, follow a similar procedure for exercise B. Have the children finish this block of exercises independently.

Adapt the procedures suggested above for the block of exercises labeled with white letters.

FUNDAMENTAL OPERATIONS

The *operations* in mathematics (addition, multiplication, and so on) are often confused with certain computational processes. In the past, arithmetic instruction emphasized the learning of *processes*, or ways of changing the names of numbers, and neglected to develop adequately the meanings for the operations. The result has been that students gained very little understanding of the operations, even though this knowledge is essential for effective problem solving.

Research suggests that children think in terms of action—that is, in terms of what they have seen happen (or have caused to happen) to blocks, markers, toys, animals, and so on. This suggestion conforms to the well-known fact that children learn through the senses of touch, sight, hearing, and smell. A modern arithmetic program should provide a wealth of experiences that direct attention to those actions that are involved in joining and separating sets of objects and that can be interpreted mathematically. In particular, the children must have experiences with those actions that are the basis for the mathematical concepts expressed by such words and symbols as "plus," "minus," "multiplied by," "divided by," "+," "−," "×," and "÷." In this section on fundamental operations, attention is given only to the actions associated with the words "plus" and "minus" and the symbols "+" and "−."

A definite sequence of learning experiences, based on sound psychological principles, must be provided before the children can successfully understand the actions associated with *plus* and *minus*. It is important to remember that, up to this point, the children have had much experience in immediate recognition of the numerousness of sets through 10. In this section, attention is focused upon the actions involved in combining one set with another set and in separating a part of a set from the whole set.

In *Book 2*, the meaning of each type of action is developed through the use of pictures. In some pictures, the objects shown are animals, mechanical toys, and so on, that appear to move of their own accord. In other

pictures, the action is portrayed by some other means—a hand movement, falling objects, and so on. As an example, the children view a picture sequence that shows two sets of sheep and then shows one set of sheep moving to join the other set of sheep. The operation of addition occurs when a child thinks of the two sets of sheep as a single set. The children also learn that a single set of objects can be separated into two subsets. In this situation, some of the objects in the original set are removed. For example, in a situation involving a set of toy drums, a hand is shown removing some of the drums. This separating action leads to the mathematical concept called subtraction.

Many experiences in combining one set of objects with another set of objects leads to the generalization that such situations are similar in the action they portray. The children learn that *plus* and "+" are associated with those action situations in which one set of objects joins (or is combined with) another set of objects. Similarly, the children formulate the generalization that *minus* and "−" are associated with those action situations in which some members of a set are removed (separated) from the original set.

The children learn to build a *phrase* to describe each joining (additive) situation. A phrase names a number and describes this number in terms of the action involved. As an example, 2 + 3 is the phrase that describes a situation in which a set of 3 objects joins a set of 2 objects. In this section, which precedes the section concerned with sums and minuends through 8, attention is focused only on the action and the symbolization of situations involving these actions. No attention is given to the numerousness of the set that results from the action because, although 2 + 3 and 5 are names for the same number, the numeral 5 does not describe the situation in terms of the action.

The children also learn to build a phrase to describe each separating (subtractive) situation. They learn, for example, that 7 − 3 is the phrase that describes a situation in which 3 objects are removed from a set of 7 objects.

The modern primary arithmetic program should help the child select from the heterogeneous experiences he has at school and at home those actions that are the basis of mathematical concepts represented by words, signs, and symbols. These experiences should be so organized that the child learns to associate a particular symbol with a particular kind of experience or set of actions. The effect of such a program of instruction on problem solving is evident. Children learn to decide whether to add or to subtract on the basis of what the situation suggests that they do with the sets involved rather than on the basis of a set of "cue" words. Problem-solving procedures become more meaningful to the children because the symbols they use are related to the action implied in the problem situation.

STA 2 page 22
Workbook edition page 87

Recognition of additive action and subtractive action; oral use of the words "plus" and "minus" [reteaching]

The children learn to recognize situations in which the action is either additive or subtractive. They use the words "plus" and "minus" to build phrases that describe the action in the situations.

Remember that a phrase names a number and describes this number in terms of the action involved. The phrase 3 + 4 tells how many objects there are and describes the action situation in which a set of 4 objects joins (or is joined with) a set of 3 objects. An important objective of this section is to help the children gain an understanding of the

symbols (+ and −) that are used to build a phrase, as well as an understanding of the phrase itself.

The words "plus" and "minus" are used orally at this time because the children should become familiar with the meaning and the sound of these words before they see them in print. The printed word and the symbol for *plus* and the printed word and the symbol for *minus* are introduced on the next page.

All the methods of illustrating action that are used in *Book 2* are shown on this page. Discuss each of these methods with the children. Ask them to explain how they can decide in what direction the animals are moving, what the action lines show (see picture C), how they can determine whether falling objects indicate a joining or a separating situation, and so on.

Discuss the pictures on this page in order. Pictures A and B each have two scenes. For the first scene in picture A, ask the children to tell how many sheep there are in each of the two sets. Next have them tell a story about what is happening in the second scene of picture A. They should observe that the set of 2 sheep is moving. The action lines show that they are moving toward the set of 3 sheep. Encourage the children to describe what is happening by saying "2 sheep are joining 3 sheep." Help them understand that "three plus two" is associated with this action situation. You might, for example, say that "three plus two" is used to tell about this action situation. Explain that "three" is the number associated with the set of sheep that is not moving, that "plus" is used because two sets are joining, and that "two" is the number associated with the set of sheep that is moving toward the set of 3 sheep. Avoid saying "three sheep plus two sheep." The word "plus" should be used *only* with numbers in expressing an additive situation. The word "minus" also is used only with numbers. Encourage the children to use the words "plus" and "minus" correctly.

Direct attention again to the second scene of picture A. Ask the children what number is associated with the set of sheep that is standing still and what number is associated with the set of sheep that is moving. Then ask the children to state again the phrase that describes the action ("three plus two"). Explain that they can also say "three plus two" in answer to the question "How many sheep are there in all?" Direct your class discussion in such a way that the children understand that the phrase 3 + 2 names a number just as the numeral 3 names a number. If a child suggests that there are "five" sheep, explain that, in this situation, "five" may also be used to answer the question "how many" but that "five" does not describe the action situation.

Now direct attention to picture B. Ask the children to name the number associated with the set of spools on the table. Next have them tell a story about what is happening in the second scene of the picture. They might say that 6 spools were placed on a table, someone bumped the table, and 2 of the spools are falling off the table. Lead the children to understand that "six minus two" describes this action situation. Call attention to the set of spools in the first scene of picture B. Emphasize that "six" is the number associated with the original set of spools. Then explain that we say "minus" to tell that some of the spools are being removed, and that "two" is the number associated with the set of spools falling off the table (or being removed). Remember to avoid saying "six spools minus two spools." The word "minus" should be used only with numbers, not with objects.

Direct attention again to the first scene of picture B. Ask the children to name the number of spools on the table. Then ask them to name the number of spools falling off the table, as shown in the second scene. Explain

that they can say "six minus two" to answer the question "How many spools are left on the table?" Accept "four" as another way of answering the question "How many spools are left on the table?" but explain that "four" does not describe the action situation.

Pictures C through H show only the action that is taking place. For each picture, the children should first decide whether the action taking place is a joining action or a separating action, and whether "plus" or "minus" is associated with this action. Then they should state the phrase that describes the number in terms of the action. Direct the discussion so that the children use the correct phrase to tell "how many" objects are in the combined set or the remaining set. As an example, for picture C the children should first decide that the action is a separating action, and that "minus" is associated with this action. Then they should state the phrase "seven minus four" to describe the situation. They should also say "There are seven minus four frogs sitting."

For those pictures in which the action is subtractive and only the action is shown (C, E, and H), it is important to remember that the children must first determine the numerousness of the original set. This must be done before they can correctly state the phrase that names a number in terms of the action involved.

One of the important objectives in the teaching of this page is to help the children understand the ideas of additive (joining) action and subtractive (separating) action and associate the appropriate term (plus or minus) with each action. Also, it is important that they understand the use of phrases as names for numbers.

STA 2 page 23
Workbook edition page 38

Additive action and subtractive action; introduction of the printed words "plus" and "minus" and the symbols "+" and "−"

The children have learned that *plus* is associated with additive action and that *minus* is associated with subtractive action. Now they learn to recognize the printed words "plus" and "minus" and the symbols "+" and "−."

Use pictures A through D to introduce the printed word "plus" and the symbol "+." Use pictures E through H to introduce the printed word "minus" and the symbol "− " As you discuss each picture, continue to call attention to the various devices (action lines, a hand, moving animals, falling objects, etc.) used to represent additive action or subtractive action.

Have the children study picture A. Then ask: "What is happening in picture A? [A hand is pushing 1 doll toward 3 dolls.] Is this joining action or separating action? [Joining action] What word do we use to tell that the picture shows joining action? [Plus] How many dolls are on the table? [3] How many dolls is the hand pushing toward the 3 dolls? [1] What phrase describes what is happening? [Three plus one] How many dolls are there in all? [Three plus one]"

Introduce the printed word "plus" and the symbol "+ " Ask various children to read the phrases given in picture A. Relate the symbols in "3 + 1" to the words in "three plus one." As you discuss picture A, emphasize that only the phrase "three plus one" describes the action situation. It may be necessary to explain that 3 + 1 and 1 + 3 do not describe the same physical situation. 3 + 1 describes a physical situation in which 1 object joins a set of 3 objects, while 1 + 3 describes a physical situation in which a set of 3 objects

joins a set of 1 object. Strive to get the children to use the proper phrase in describing an action situation.

Use a similar procedure for each of pictures B, C, and D. For each of pictures C and D, ask the children to name the numeral that replaces each placeholder. Be sure the children give the phrase that correctly describes the situation.

Then direct attention to picture E. It is important to remember that, for pictures that show subtractive situations, the children must first determine the numerousness of the original set before they can give the phrase that describes the action. Ask: "How many rabbits were there to begin with? What is happening in the picture? How many rabbits are running away? What word do we use to tell that the action is a separating action? What phrase describes the situation? Does 5 – 2 tell you how many rabbits are not running away?"

Introduce the printed word "minus" and the symbol "–." Ask various children to read each phrase given in picture E. Relate the symbols in "5 – 2" to the words in "five minus two." Help them to understand that "five" is the number associated with the original set, that "minus" tells that some members of the set are being removed, and that "two" is the number associated with the set being removed.

Use a similar procedure for each of pictures F, G, and H. For each of pictures G and H, ask the children to replace each placeholder with a numeral that completes correctly the phrase that describes the action in the picture.

STA 2 page 24

Distinguishing between additive action and subtractive action

The children first decide whether one set is joining the other set (additive action) or whether some members of a set are moving away (subtractive action). Then they determine whether "plus" or "minus" is associated with this action and build the phrase that names a number in terms of the action situation.

You may wish to provide each child with a set of 8 markers or similar objects and a set of 15 cards. One of the following should be written on each card: the numerals from 1 through 8, the word and symbol for *plus*, the word and the symbol for *minus*, the square screen (■), the round screen (◍), and the wavy line (〰).

First discuss each picture in terms of the action shown. For each picture, have the children decide which word and symbol are associated with the action. Have them identify the word and the symbol in the list at the top of the page. (The colored mark under each word and symbol can be used as a means of identification.) For picture A, the children should decide that one set is joining the other set and that the word "plus" (red mark) and the symbol "+" (green mark) can be associated with the action. They could also respond by holding up the cards that show the correct word and symbol.

After discussing each picture in terms of the action, ask the children to state the phrase that names the number of objects and describes this number in terms of the action involved. Remember that, in the pictures that show subtractive action, the children must first determine the numerousness of the original set before they can decide what phrase corresponds to the situation.

As you discuss picture A with the children, ask questions that lead them to conclude that "one plus two" describes what is happening in the picture. Call attention to the placeholders in the picture. Ask the children to use their cards to build the phrase that describes the action in the picture, using the correct numerals and symbol as replacements for the placeholders. You might have various children state the numeral that replaces the square screen, the numeral that replaces the

round screen, and the symbol (identified by the colored mark) that replaces the wavy line. Use a similar procedure for each of the other pictures on this page. Let the children use markers to illustrate each picture.

Put other phrases, such as 3 + 4, six plus two, 8 − 3, and five minus two, on the chalkboard. Have the children use their markers to illustrate the action situation described by each phrase.

Practice tablet page 19
Workbook edition page 39

Call attention to Picture A. Ask a child to tell whether a joining action or a separating action is shown. Ask which word, *plus* or *minus*, describes this action. Tell the children to draw a ring around this word. Ask a child to tell how many squirrels there are in all in a way that also tells what is happening. Tell the children to write "1 + 3" on the response space.

If necessary, follow a similar procedure for Picture B. Then have the children complete the page. For each picture, they first decide whether a joining action or separating action is shown. Then they draw a ring around the appropriate word. Finally, they write on the response space the phrase that describes what is happening.

Practice tablet page 20
Workbook edition page 40

For each picture, the children first decide whether a joining action or a separating action is shown. Then they draw a ring around the appropriate word. Finally, they write on the response space the phrase that describes what is happening.

STA 2 page 25
Workbook edition page 41

The commutative property of addition; introduction of the symbol for "is equal to" [reteaching]

The children learn that because certain phrases, such as 3 + 1 and 1 + 3, express the same number, they can say that the numbers are equal. They also learn that "=" is the symbol for "is equal to."

The commutative property of addition deals with the order in which we think about numbers that are added. The property states that if a first number, such as 1, is added to a second number, such as 3, the result is the same as if the second number (3) is added to the first number (1). 3 + 1 is equal to 1 + 3. Note, however, that the commutative property does not apply to subtraction; 5 − 3 does not equal 3 − 5.

Notice that picture A consists of two parts. The first part shows 1 duck running toward 3 ducks. The phrase 3 + 1 describes this action situation and tells how many ducks there are. The second part shows 3 squirrels running toward 1 squirrel. Here, the phrase 1 + 3 describes the action and tells how many squirrels there are. Observe that a one-to-one correspondence can be set up between the two sets of objects. Since there is one duck for each squirrel and one squirrel for each duck, the sets have the same number of members. Therefore, the number of ducks (3 + 1) is equal to the number of squirrels (1 + 3), or 3 + 1 = 1 + 3. By generalizing from this and similar situations, the children learn that the order in which two numbers are added does not affect the result.

It is important that the children begin to understand the meaning of the commutative property. However, it is not important or necessary for them to be familiar with the words "commutative property of addition."

Direct attention to picture A. Have the children describe what they see in each part. Then have them name the phrase that describes each of the additive situations pictured. Direct the class discussion in such a way that the children will clearly understand that the situation pictured in the first part is different from that pictured in the second part. Draw attention to the phrase "3 + 1." Ask the children to explain why they cannot use this phrase to describe the picture that shows three squirrels running to join one squirrel. Then ask them to explain why they cannot use "1 + 3" to describe the action shown in the first part of picture A.

Next, have them trace the "lines" and determine that there is a squirrel for each duck and that there is a duck for each squirrel. Ask: "Are there as many squirrels as ducks? Are there as many ducks as squirrels?" Have the children read aloud the sentence "3 + 1 is equal to 1 + 3." Help the children understand that the words "is equal to" are used because the number of ducks is the *same* as the number of squirrels.

Finally, introduce the symbol "=." Explain that this symbol may be read either as "is equal to" or as "equals." The children should understand that "equals" is a shortened form of "is equal to." Then have them read aloud the sentence "3 + 1 = 1 + 3."

Follow a similar procedure for pictures B, C, and D. For pictures C and D, the children should name the numeral that replaces each placeholder. For picture C, continue class discussion until you are satisfied that the children understand that the phrase that describes the action in the second part is "4 + 3." For picture D, be sure the children understand that the numeral 6 replaces the square screen and the numeral 2 replaces the round screen.

STA 2 page 26
Workbook edition page 42

The commutative property of addition [reteaching]

The children continue to study the commutative property of addition.

For each of pictures A through D, have the children describe what is happening in each part and state the phrase that names a number in terms of the action situation. Ask questions to help them discover that, for each picture, the number of objects shown in the first part is equal to the number of objects shown in the second part. Then have them read the sentence and decide what numeral replaces each placeholder. Have several children read the sentence aloud, making the correct replacement (or replacements).

Next direct attention to the exercises at the bottom of the page. For exercise A, have the children first read the sentence silently. Ask them to imagine an action situation represented by 1 + 2. Have various children describe the situation they are thinking of. Be sure that each child describes a situation in which a set of 2 objects is joining a set of 1 object. Have them use markers or other small objects to illustrate the situation.

Then ask the children to suggest a replacement for the placeholder in ■ + 1. Explain that the replacement must result in a phrase that will name the same number as 1 + 2. Ask them to imagine and describe a situation associated with 2 + 1. Have them use markers to illustrate the situation. Be sure that the situation described suggests a set of 1 object joining a set of 2 objects. Finally, have various children state the completed sentence "1 + 2 = 2 + 1."

Use a similar procedure for each of exercises B through P. For exercises Q through X, the children make two replacements in each sentence. Be sure the children understand

that the numbers associated with each of the sets in the second additive situation must be the same as the numbers associated with each of the sets in the first additive situation. As an example, for exercise Q, you should accept only "1 + 4 = 4 + 1" as the completed sentence.

You may wish to use the exercises for written practice. If so, have the children write the letters A through X on their papers. Then have them read each sentence silently, decide what numeral replaces each placeholder, and write the completed mathematical sentence. For exercise A, they should write:

A 1 + 2 = 2 + 1

Practice tablet page 21
Workbook edition page 43

Call attention to Picture A. Say: "There are the same number of cups as boats. We want to find two different names for this number. Look at the cups. Give a name for the number of cups. This name should tell what is happening. [Be sure to get 1 + 2 as a response.] Find the response space that is under the cups. Write 1 + 2 on this response space. Now look at the boats. Remember that there are as many boats as cups. Give a name for the number of boats. This name should tell what is happening. [Be sure to get 2 + 1 as a response.] Find the response space that is under the boats. Write 2 + 1 on this response space. We have found two names for the same number. What are these two names?"

Have the children complete the page. Remind them that the same number of objects is shown in each part of a picture and that, for each picture, they are to find two names for the same number.

Practice tablet page 22
Workbook edition page 44

Call attention to Picture A. Say: "There are the same number of flowerpots as clothespins. We want to find two different names for this number. Look at the flowerpots. Give a name for the number of flowerpots. This name should tell what is happening. [Be sure to get 4 + 2 as a response.] Find the response space that is under the flowerpots. Write 4 + 2 on this response space. Now look at the clothespins. Remember that there are as many clothespins as flowerpots. Give a name for the number of clothespins. This name should tell what is happening. [Be sure to get 2 + 4 as a response.] Find the response space that is under the clothespins. Write 2 + 4 on this response space. We have found two names for the same number. What are these two names?"

Have the children complete the page. Remind them that the same number of objects is shown in each part of a picture and that, for each picture, they are to find two names for the same number.

STA 2 page 27

Keeping skillful: closed curves, polygons, and segments; ideas of "one greater than," "one less than," and betweenness for numbers

The children practice what they have learned about closed curves, polygons, and segments. They also give attention to the ideas of "one greater than," "one less than," and betweenness for numbers.

The heading "Keeping skillful" is given to this page. Pages that have this heading are designed to give the children additional practice, maintenance, and review of the concepts and skills they have studied previously in the book. The exercises on these pages may be used either for written practice or for oral prac-

tice. Throughout *Seeing Through Arithmetic 2*, the "Keeping skillful" exercises appear on pages that have a yellow background.

For each of pictures A through F, first have the children decide what kind of curve is represented. They should identify the paths in A, C, D, and E as closed curves, and the paths in B and F as segments. They should also decide that the paths in A, C, and E represent polygons.

Ask the children to identify dots for points that are inside or outside each closed curve. For the paths in pictures A, C, D, and E, ask questions that will help them recall that they cannot say that one point is between other points in a closed curve. Ask them to name the sides and vertices of each polygon.

For the paths in pictures B and F, ask the children to name various segments. For each segment named, ask them to identify the endpoints of the segment. Then ask questions that will help the children recall that they can identify points that are between other points in a segment. Have them name such points. Also have them name points that are not in the segment. Finally, ask questions to help them recall that they cannot say that a point is inside or outside a curve that is a segment.

The exercises A through L that are identified by green letters review the ideas of "one greater than" and "one less than." The exercises A through L that are identified by red letters review the idea of betweenness for numbers. These two blocks of exercises may be used for oral practice or for written practice. If the exercises are used for oral practice, have the children read each exercise silently and decide what numeral correctly replaces the square screen. Then have various children read the sentence orally, supplying a replacement for the placeholder. Ask other children to verify the responses given.

If the exercises are used for written practice, have the children write the letters included in each block. Then have them read each exercise and write their response beside the appropriate letter.

BASIC FACTS

Although computation usually arises out of problem situations and is seldom an end in itself, children must be carefully taught computational skills from the beginning. To be successful in problem solving, children must learn the basic facts for addition, subtraction, multiplication, and division. In the *Seeing Through Arithmetic* program, they begin their study of the basic facts with the basic facts for addition and subtraction.

The teaching of these basic facts must rest upon a foundation established previously. First the children learn the meaning of the numbers through 10. Next they learn to recognize joining situations (additive action) and separating situations (subtractive action) and to build phrases that name a number and describe this number in terms of the action situation. After these experiences, they are ready to learn the *standard name* for the number identified by each phrase. A numeral is the standard name for a number if it occurs in the list 0, 1, 2, and so on. Thus, while $7 - 3$ names a number and describes this number in terms of the action situation, the standard name of this number is 4. Note that the standard name does not describe the number in terms of the action involved. In the same way, 7 and $3 + 4$ are names for the same number: 7 is the standard name for the number; $3 + 4$ describes the number in terms of the action involved.

In the addition basic fact $2 + 3 = 5$, 2 and 3 are called *addends* and 5 is generally called the *sum*. Strictly speaking, however, $2 + 3$ is also the *sum* of 2 and 3; 5 is the standard name for this sum. In the subtraction basic fact $7 - 3 = 4$, 7 is the *minuend*, 3 is the

subtrahend, and 4 is the *difference.* It is not necessary to use these terms in talking with the children, nor should you expect the children to associate these terms with specific numbers in addition or subtraction basic facts. At the elementary level, the children should understand the ideas involved without being burdened with the task of learning the terms.

The basic facts are organized in such a way that the relationship between addition and subtraction is emphasized. Addition is considered a "doing" operation, while subtraction is an "undoing" operation. Consider 3 + 4 = 7 and 7 − 4 = 3. The mathematical sentence 3 + 4 = 7 is associated with a set of 4 objects joining a set of 3 objects to form a set of 7 objects. The mathematical sentence 7 − 4 = 3 is associated with a set of 4 objects being removed from the set of 7 objects to obtain a set of 3 objects. Although this relationship is inherent in the presentation of the basic facts for each number, attention is not focused directly on the relationship until after the basic facts through 8 have been presented.

In *Book 1* of the *STA Program,* the first presentation of basic facts includes only those addition facts in which neither addend is greater than 4 and only those subtraction facts in which neither the subtrahend nor the difference is greater than 4. The second presentation, entitled "Extension," includes the remaining basic facts with sums and minuends of 6, 7, and 8. In *Book 2,* the basic facts with sums and minuends through 8 are retaught first. Then the facts with sums and minuends of 9 and 10 are taught. After the principles of the numeration system have been presented, the basic facts with sums and minuends of 11 and 12 are introduced. Then multiplicative action and divisive action are introduced, and the multiplication and division basic facts with products and dividends through 12 are taught. Thus, 124 addition and subtraction basic facts and 24 multiplication and division basic facts are included in the basic program.

The presentation of all addition and subtraction basic facts (sums and minuends through 18) is completed in the "Extension" section. (The material presented in the "Extension" section is a part of the *Seeing Through Arithmetic* plan for providing for individual differences.) The multiplication and division basic facts with products and dividends through 18 also are presented in this section. Thus, in this section, 42 addition and subtraction basic facts and 22 multiplication and division basic facts are introduced.

It seems advisable to discuss here whether a basic fact should be expressed in the horizontal form (a mathematical sentence) or the vertical form (as in computation). The vertical arrangement of numerals is a convenient device for computation, but it has no special advantage for expressing basic facts. It is important to remember that in grades 1 and 2 the emphasis should be upon the meaning of addition, subtraction, and so on, in terms of the actions with which they are associated. It is these actions that lead to the concepts that are symbolized by special symbols (+, −, and so on) written between the numerals that represent the numerousness of the sets. The various possible arrangements of numerals for computational purposes can best be presented to children when they are being taught how to compute. However, since they may see basic facts expressed vertically in published tests or in other materials, they should be familiar with this form. For this reason, the vertical arrangement may be used occasionally as a part of classroom activities. It is worthy of notice that, in other countries of the world, sentences are usually used to express basic facts.

STA 2 page 28
Workbook edition page 45

Basic facts: sums of 5 [reteaching]

The children have learned that a phrase names a number and describes this number in terms of the action involved. Now they learn what is meant by *standard names* for numbers. They learn that the numeral 5 is a standard name for a number. The number 5 is also named by such phrases as 3 + 2, 1 + 4, 2 + 3, and 4 + 1.

Notice that the first scene of each picture shows the action that is taking place. The second scene shows the set that results from the action.

Direct attention to the first scene in picture A. Have the children describe what is happening and give the phrase that names the number of objects and describes this number in terms of the action involved. Then ask the children if they can give a name for the number of objects in the second scene. You should expect them to identify the numerousness of the set shown in the second scene as 5. They should recognize the numerousness of the set by regrouping the objects, not by counting. Ask the children if there are as many dogs in the first scene as there are in the second. Explain that 3 + 2 and 5 are names for the same number and that they can say "3 + 2 = 5." 3 + 2 = 5 is the *sentence* that describes the situation pictured in A. Have the children read the sentence given in picture A. Explain that 5 is the standard name for the number. You may wish to provide each child with a set of 5 markers or other small objects that he can use at his desk after the page has been discussed.

Follow a similar procedure for pictures B, C, and D. For each of pictures E and F, first have the children describe what is happening. Then have them give the numeral that replaces each placeholder. Ask various children to read the sentences aloud, supplying the appropriate numerals. Have the class check to see that the sentence is read correctly.

It is best to avoid the use of such expressions as "3 dogs plus 2 dogs is equal to 5 dogs." "Plus" and "is equal to" are used only with numbers. If you wish to talk about the objects in the picture, you can say "Two dogs are joining three dogs. Now there are five dogs."

STA 2 page 29
Workbook edition page 46

Basic facts: minuends of 5 [reteaching]

The children now learn the standard name for numbers named by the phrases 5 - 2, 5 - 3, 5 - 4, and 5 - 1.

Notice that the first scene of each picture shows the action, and the second scene shows the set resulting from the action. It is important to keep in mind that, in a subtractive situation, the children must first determine the numerousness of the original set.

Direct attention to the first scene in picture A. Ask how many umbrellas there were on the table to begin with. Then have the children tell what is happening and give the phrase that is associated with the pictured situation. Encourage the children to discover that the set of umbrellas shown in the second scene is the set that remains after the action pictured in the first scene has been completed. Ask how many umbrellas are shown in the second scene. You should expect the children to recognize the number of objects in the set without counting. Ask them to tell in what two ways they can name the number of umbrellas that remain. Encourage them to give the standard name, 3, and the phrase, 5 - 2. In class discussion help the children understand that, since 5 - 2 and 3 are names for the same number, they can say "5 - 2 = 3." Then have various children read aloud the sentence below the picture.

Use a similar procedure for each of pictures B, C, and D. For each of pictures E and F, have the children give the phrase and the standard name and then have them decide what numerals replace the square and round screens in the sentence printed in the picture. Then have various children read the sentence aloud, supplying the appropriate numerals.

It is best to avoid the use of such expressions as "5 umbrellas minus 2 umbrellas is equal to 3 umbrellas." "Minus" and "is equal to" should be used only with numbers.

STA 2 page 30

Basic facts: sums and minuends of 5 [reteaching]

The children apply what they have learned about sums and minuends of 5.

Notice that single pictures, rather than two-scene pictures, are used on this page. For the pictures that show additive action, the children should observe the joining action and determine the numerousness of the resulting set. For the pictures that show subtractive action, they should observe the separating action and determine the numerousness of the original set. Then, for each picture, they should give the sentence that describes the situation. The children apply what they have learned about sums and minuends of 5 to build a sentence that corresponds to each pictured situation.

Notice also that picture I is the first instance of a picture of an action situation in which the objects in one set differ from the objects in a second set. However, the kind or kinds of objects are not important. Rather, it is the number associated with each given set that is important.

If possible, provide each child with six or eight small objects to use as markers and a set of cards, each card showing one of the following symbols: +, −, =, 1, 2, 3, 4, 5, 6, 7, 8, 9. The markers can be used to provide the children with concrete experiences involving additive and subtractive situations after the page has been discussed. The cards may then be used to build sentences associated with the action.

Direct attention to picture A. Ask the children how many bears there were to begin with. Then ask them to describe what is happening. The children should say that 3 bears are moving away from 2 bears. Next ask what word corresponds to the action. Have various children give the sentence associated with the action. Follow a similar procedure for each of the other pictures.

After all the pictures on page 30 have been discussed, direct attention to the exercises at the left. For each sentence, ask the children to read the exercise silently and determine the numeral that replaces the screen. Then have one child read the sentence aloud, supplying the correct numeral. Ask other children to verify the reading or to read aloud the sentence if they think an error was made.

Finally, for each sentence, have the children identify, if possible, a picture on page 30 that is associated with the sentence (picture D and sentence A, for example). It is not possible to find a picture for each sentence. For example, none of the pictures on this page can be associated with 3 + 2 = ▥.

For practice, you can select pictures at random and have the children find all the sentences at the left that are associated with each pictured situation. As an example, for picture C, the children should give sentence B, sentence I, and sentence W. Direct class discussion to ensure the children understand that the sentence 2 + 3 = 5 and the sentence 3 + 2 = 5 are associated with different physical situations.

Practice tablet page 23
Workbook edition page 47

Call attention to Picture A. Say: "Give a name for the number of dogs in a way that

tells what is happening. [Be sure to get 3 + 2 as a response.] What is the standard name for 3 + 2? Now give the arithmetic sentence that tells what is happening in this picture. Write 3 + 2 = 5 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

Point out that some pictures show objects that are moving away. Be sure that the children understand that they will use the symbol for minus in the sentence that describes what is happening in such a picture.

After the children have completed the pictures, direct attention to Exercises A through P. Say: "Look at Exercise A. What is the standard name for 5 - 1? Write 4 on the response space. For each of the other exercises, write on the response space the standard name for the number."

Practice tablet page 24
Workbook edition page 48

Call attention to Picture A. Say: "Give a name for the number of tigers left in a way that tells what is happening. [Be sure to get 5 - 4 as a response.] What is the standard name for 5 - 4? Now give the arithmetic sentence that tells what is happening in this picture. Write 5 - 4 = 1 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

Point out that some pictures show objects that are joining other objects. Be sure the children understand that they will use the symbol for plus in the sentence that describes what is happening in such a picture.

After the children have completed the pictures, direct attention to Exercises A through P. Say: "Look at Exercise A. What is the standard name for 2 + 3? Write 5 on the response space. For each of the other exercises, write on the response space the standard name for the number."

STA 2 page 31
Workbook edition page 49

Basic facts: sums and minuends of 4 [reteaching]

The children learn the standard name for the number named by the phrases 2 + 2, 3 + 1, and 1 + 3. They also learn the standard names for numbers named by 4 - 2, 4 - 1, and 4 - 3.

Direct attention to the first scene of picture A. Ask the children to describe what is happening. They should say that 2 tigers are moving toward 2 other tigers. Have them give the phrase (2 + 2) that names the number of tigers in terms of the action. Ask the children if they can give another name for the number of tigers shown in the second scene. You should expect them to identify the numerousness of the group as 4.

Ask the children if there are as many tigers in the second scene as in the first scene. Then ask if 2 + 2 and 4 are names for the same number. Direct the discussion to help the children understand that, since 2 + 2 and 4 are names for the same number, they can say "2 + 2 = 4." Emphasize that 4 is a standard name and that 2 + 2 is a phrase. The standard name "4" and the phrase "2 + 2" name the same number, but the numeral "4" does not describe the number in terms of the action. Ask various children to read aloud the sentence 2 + 2 = 4.

For the first scene of picture B, have the children describe what is happening. Ask them to give the phrase that names the number of spools that are left. Direct questions that will help them recall that for a separating situation the second scene shows the set of spools that is left after the action shown in the first scene is completed. Ask the children to give the standard name for the number of spools that are left. Then ask if "4 - 2" and "2" are names for the same number. Ex-

plain that, since "4 − 2" and "2" are names for the same number, they can say "4 − 2 = 2." Have various children read aloud the sentence 4 − 2 = 2.

For pictures C and D, adapt the procedures outlined for picture A. For pictures E and F, adapt the procedures suggested for picture B.

The children can use their markers and cards as each picture is discussed, or you can have them use markers and cards after completing the discussion of the page.

STA 2 page 32

Basic facts: sums and minuends of 4 and 5 [reteaching]

The children apply what they have learned about sums and minuends of 4 and 5 to determine what numerals replace the screens in sentences that express basic facts.

The pictures and the sentences on page 32 include only those basic facts that the children have studied previously. The pictures may be used in any order. For each picture, have the children give the complete sentence that can be associated with the situation pictured. Note the unlike objects shown in picture I. After discussing all the pictures, have the children complete the sentences that are labeled with red letters.

The teaching suggestions given for using *STA 2* page 30 may be adapted to this page.

Practice tablet page 25
Workbook edition page 50

Call attention to Picture A. Say: "Give a name for the number of cups that are left in a way that tells what is happening. [Be sure to get 4 − 2 as a response.] What is the standard name for 4 − 2? Now give the arithmetic sentence that tells what is happening in this picture. Write 4 − 2 = 2 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

After the children have completed the pictures, direct attention to Exercises A through U. Say: "Look at Exercise A. What is the standard name for 3 + 1? Write 4 on the response space. For each of the other exercises, write the standard name for the number named in the exercise."

STA 2 page 33
Workbook edition page 51

Basic facts: sums and minuends of 2 and 3 [reteaching]

The children learn standard names for the numbers named by the phrases 1 + 1, 2 − 1, 2 + 1, 1 + 2, 3 − 1, and 3 − 2.

Some of the pictures show additive situations in which the set resulting from the action is a set of 3 or a set of 2. Other pictures show subtractive situations in which the original set is a set of 3 or a set of 2.

The teaching suggestions given for using *STA 2* pages 28, 29, and 31 (Wkbk pages 45, 46, and 49) may be adapted to this page.

STA 2 page 34

Basic facts: sums and minuends through 5 [reteaching]

The children continue to apply what they have learned about basic facts for numbers from 2 through 5.

The pictures and the sentences on page 34 deal only with basic facts that the children have been studying. For each picture, have the children give the completed sentence that can be associated with the situation pictured. After discussing each picture, have the children take turns in reading the sentences labeled with blue letters. For each sentence,

have them give the numeral that replaces the screen.

The teaching suggestions given for using *STA 2* page 30 may be adapted to this page.

Practice tablet page 26
Workbook edition page 52

Call attention to Picture A. Say: "Give a name for the number of hats in a way that tells what is happening. [Be sure to get 1 + 1 as a response.] What is the standard name for 1 + 1? Now give the arithmetic sentence that tells what is happening in this picture. Write 1 + 1 = 2 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

After the children have completed the pictures, direct attention to Exercises A through U. Say: "Look at Exercise A. What is the standard name for 3 - 2? Write 1 on the response space. For each of the other exercises, write the standard name for the number named in the exercise."

Practice tablet page 27
Workbook edition page 53

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 5 - 1 = 4 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

After the children have completed the pictures, direct attention to Exercises A through V. Say: "For each exercise, you are to write on the response space the standard name for the number named in the exercise."

Practice tablet page 28
Workbook edition page 54

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 4 + 1 = 5 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

After the children have completed the pictures, direct attention to Exercises A through V. Say: "For each exercise, you are to write on the response space the standard name for the number named in the exercise."

STA 2 page 35

Keeping skillful: ideas of "greater than," "less than," and "equal to"; addition and subtraction basic facts

The children review the ideas of "greater than," "less than," and "equal to." They also review the basic facts with sums and minuends of 2 through 5.

This page can be used to evaluate class or individual progress. The responses a child gives for the exercises on this page will indicate how well he understands the ideas of "greater than," "less than," and "equal to," and to what degree he has mastered the basic facts taught up to this point.

The responses to the exercises can be given orally, or they can be written. If the responses are to be written, be sure the children understand how you want the responses recorded. You might want to duplicate answer sheets and have the children indicate their responses on these. Or you might want the children to list the letters for each block of exercises and write their responses beside the appropriate letters.

If the responses are given orally, for each sentence first have the children read the sentence silently and determine what replacement correctly completes the sentence. Then

ask one child to read the sentence aloud as he thinks it should be completed. Have the other children verify the response and suggest corrections if they think an error has been made.

STA 2 page 36
Workbook edition page 55

Basic facts: sums and minuends of 6 [extension]

The children begin their study of the addition and subtraction basic facts for the number 6.

Discuss the pictures in order. Call attention to picture A. Ask the children to describe what is happening in the first scene, then to give the phrase that names the number of dogs and describes this number in terms of the action involved. Have the children tell how many dogs are in the second scene. Ask them if the same number of dogs is shown in each scene, and if the numeral 6 is the standard name for the number named by 3 + 3. Direct questions to remind the children that they can say "3 + 3 = 6" because 3 + 3 and 6 are names for the same number. Have various children read aloud the sentence given in picture A. Follow a similar procedure for each of the other pictures.

The teaching suggestions given for using *STA 2* pages 28, 29, and 31 (Wkbk pages 45, 46, and 49) may be adapted to this page.

STA 2 page 37
Workbook edition page 56
Basic facts: sums and minuends of 6 [extension]

The children continue to study the addition and subtraction basic facts for the number 6.

The teaching suggestions given for using *STA 2* pages 28, 29, and 31 (Wkbk pages 45, 46, and 49) may be adapted to this page.

STA 2 page 38
Basic facts: sums and minuends through 6 [extension]

The children apply what they have learned about sums and minuends of 2 through 6.

The teaching suggestions given for using *STA 2* page 30 may be adapted to this page.

Practice tablet page 29
Workbook edition page 57

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 4 + 2 = 6 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture.

After the children have completed the pictures, direct attention to Exercises A through U. Say: "For each exercise, you are to write on the response space the standard name for the number named in the exercise."

Practice tablet page 30
Workbook edition page 58

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 6 - 1 = 5 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture.

After the children have completed the pictures, direct attention to Exercises A through V. Say: "For each exercise, you are to write on the response space the standard name for the number named in the exercise."

STA 2 page 39
Workbook edition page 59
Basic facts: sums and minuends of 7 [extension]

The children begin their study of the addition and subtraction basic facts for the number 7.

The teaching suggestions given for using *STA 2* pages 28, 29, and 31 (Wkbk pages 45, 46, and 49) may be adapted to this page.

STA 2 page 40
Workbook edition page 60
Basic facts: sums and minuends of 7 [extension]

The children continue to study the basic facts for the number 7.

The teaching suggestions given for using *STA 2* pages 28, 29, and 31 (Wkbk pages 45, 46, and 49) may be adapted to this page.

STA 2 page 41

Basic facts: sums and minuends of 7 [extension]

The children have studied all the basic facts with sums and minuends of 7. Now they practice and apply what they have studied. They determine the numerals that replace the placeholders in sentences that express basic facts.

Notice that on this page each picture illustrates a different basic fact. For each picture, the children should describe the action as either joining action (additive) or separating action (subtractive). Then, for a picture that shows joining action they should determine the numerousness of the resulting set, and for a picture that shows separating action they should determine the numerousness of the original set. Finally, they should give a sentence that is associated with the situation pictured.

Direct attention to picture A. Ask the children to describe what is happening. They should say that a set of 3 objects is being joined to a set of 4 objects. Next ask them which word, "plus" or "minus," is associated with the action. Then have them give a sentence associated with the pictured situation. For picture A, they should give the sentence "Four plus three is equal to seven." Follow a similar procedure for each of the other pictures.

After each picture has been discussed, direct attention to the sentences printed in the pictures. For each picture, have the children read the sentence silently and decide what numerals replace each square or round screen. Then ask various children to read aloud the sentence, supplying the appropriate replacements. The other children can verify the reading or correct errors that are made.

After discussing the page, you could have the children use markers or small objects to reproduce the action shown in the pictures. The children could also use cards, each with a numeral, a symbol, or a word printed on it, to build mathematical sentences for the various situations.

STA 2 page 42
Basic facts: sums and minuends through 7 [extension]

The children apply what they have learned about the addition and subtraction basic facts for numbers from 2 through 7.

The teaching suggestions given for using *STA 2* page 30 may be adapted to this page.

Practice tablet page 31
Workbook edition page 61

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 7 − 5 = 2 on the response space. For each of the other

pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

After the children have completed the pictures, direct attention to Exercises A through U. Say: "For each exercise, you are to write on the response space the standard name for the number named in the exercise."

Practice tablet page 32
Workbook edition page 62

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 3 + 3 = 6 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

After the children have completed the pictures, direct attention to Exercises A through V. Say: "For each exercise, you are to write on the response space the standard name for the number named in the exercise."

STA 2 page 43
Workbook edition page 63
Basic facts: sums and minuends of 8 [extension]

The children begin to study the basic facts with sums and minuends of 8.

The teaching suggestions given for using *STA 2* pages 28, 29, and 31 (Wkbk pages 45, 46, and 49) may be adapted to this page.

STA 2 page 44
Workbook edition page 64
Basic facts: sums and minuends of 8 [extension]

The children extend their study of the basic facts for the number 8.

The teaching suggestions given for using *STA 2* pages 28, 29, and 31 (Wkbk pages 45, 46, and 49) may be adapted to this page.

STA 2 page 45
Workbook edition page 65
Basic facts: sums and minuends of 8 [extension]

The children complete their study of the addition and subtraction basic facts for the number 8. They also practice and apply what they have learned.

The teaching suggestions given for using *STA 2* pages 28, 29, and 31 (Wkbk pages 45, 46, and 49) and *STA 2* page 41 may be adapted to this page.

STA 2 page 46
Basic facts: sums and minuends of 8 [extension]

The children continue to practice and apply what they have learned concerning basic facts with sums and minuends of 8.

The teaching suggestions given for using *STA 2* page 41 may be adapted to this page.

STA 2 page 47
Basic facts: sums and minuends through 8 [extension]

The children continue to apply what they have learned about the basic facts from 2 through 8.

The teaching suggestions given for using *STA 2* page 30 may be adapted to this page.

Practice tablet page 33
Workbook edition page 66

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 2 + 6 = 8 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

After the children have completed the pictures, direct attention to Exercises A

through U. Say: "For each exercise, you are to write on the response space the standard name for the number named in the exercise."

Practice tablet page 34
Workbook edition page 67

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 8 − 5 = 3 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

After the children have completed the pictures, direct attention to Exercises A through V. Say: "For each exercise, you are to write on the response space the standard name for the number named in the exercise."

Practice tablet page 35
Workbook edition page 68

Notice that this page consists of two blocks of exercises with two columns of exercises in each block. Use your own judgment as to how many of these exercises you can assign at one time.

Tell the children that they are to complete each exercise by writing the correct numeral on the response space.

Workbook edition page 69

See the directions given for *STA 2* page 35.

FUNDAMENTAL OPERATIONS

In this section under the heading "Fundamental Operations," several basic ideas are emphasized: names for the same number; ideas of "greater than," "less than," and "equal to"; and relationship between addition and subtraction. The unified study presented here will strengthen the children's understanding of these basic ideas.

STA 2 pages 48 and 49
Workbook edition pages 70 and 71

Names for the same number [extension]

The children have learned to name a number by using either a standard name or a mathematical phrase. Now they see all the phrases they have studied that are names for the numbers from 1 through 8.

Direct attention to pictures A through F. For each picture, ask the children to describe what they see. Then ask them to give the phrase associated with the situation in the picture. When you have completed the discussion of a picture, write the phrase given for the picture on the chalkboard. The phrases for pictures A through F are: 3 + 3, 1 + 5, 7 − 1, 2 + 4, 8 − 2, and 5 + 1.

Direct attention to the phrases you have listed on the chalkboard. Have the children give the standard name for the number named by each phrase. As they do this, write the numeral "6" opposite each phrase that you have listed on the chalkboard. Direct questions to help the children discover that each of the phrases names the number 6.

Next, call attention to the standard names and phrases listed at the bottom of these two pages. Ask the children to find the standard name for the number 6. Direct attention to the phrases listed below the numeral "6." Emphasize that they have studied all these different names for the number 6. Ask them to find in their books the phrase that is not listed on the chalkboard. They should identify "4 + 2" as the phrase not listed.

Direct attention to pictures G through L. For each picture, have the children describe what they see and give a phrase associated with the situation in the picture. Then ask them to give the standard name for the number

named by the phrase. (The phrases and standard names for pictures G through L are: 8 − 6 and 2; 6 + 2 and 8; 7 − 4 and 3; 7 − 2 and 5; 4 + 3 and 7; and 8 − 4 and 4.) Write on the chalkboard the phrase and the standard name. Ask the children to find, at the bottom of the two pages, the standard name and the phrases associated with picture G. Have various children read the phrases listed under the numeral "2." Write the phrases on the board and emphasize that they have studied all these different names for the number 2. Discuss pictures H through L in the same way.

You may want to have the children use markers to illustrate the phrases listed under each of the standard names, or you may want to have them describe physical situations that can be related to each phrase.

STA 2 page 50
Workbook edition page 72

Ideas of "greater than," "less than," and "equal to" [extension]

The children learn to use phrases in expressing relationships between numbers.

Direct attention to the first part of picture A. Have the children describe what is happening. Then ask them to give the phrase (1 + 2) that tells how many brown blocks (Wkbk, gray) there are. Next direct attention to the second part and have the children describe what is happening. Ask them to give the phrase (5 − 2) that tells how many blue blocks (Wkbk, orange) there are in the remaining set. Direct questions in such a way that the children understand that there are as many brown blocks (Wkbk, gray) as there are blue blocks (Wkbk, orange) after two blue blocks are removed.

Have the children give the standard name of the number named by the phrases "1 + 2" and "5 − 2." Then ask the children if these phrases are names for the same number. Ask a child to read the sentence in picture A aloud. Direct questions that will help the children understand that the sentence "1 + 2 is equal to 5 − 2" is true. Then discuss picture B in a similar way.

For picture C, have the children observe what is happening and give the phrase associated with each part of the picture ("8 − 3" and "1 + 3"). Ask them if there are as many cars in the set that remains as there are tractors in the combined set. Continue with questions until they understand that there are more cars in the remaining set than there are tractors in the combined set.

Next, ask the children to give the standard names for the numbers named by 8 − 3 and 1 + 3. Ask if 5 is greater than 4. Direct questions that will help the children understand that the sentence "8 − 3 *is greater than* 1 + 3" is true. They also should understand that because there are fewer tractors in the combined set than cars in the remaining set, the sentence "1 + 3 *is less than* 8 − 3" is true.

Emphasize that "more than" and "fewer than" are used when talking about objects; "greater" and "less" are used when comparing numbers.

Use similar procedures for pictures D through F. Notice the wavy lines in pictures E and F. The children should read each sentence and replace the wavy line with the group of words that correctly completes the sentence.

When you have discussed each picture, you could have the children use markers to illustrate the situation in each part of a picture. Use the ideas of one-to-one correspondence and equivalent sets to reinforce the ideas presented on this page.

Practice tablet page 36
Workbook edition page 73

Call attention to Picture A. Say: "Look at the ducks. Give a name for the number of

ducks. This name should tell what is happening. [4 + 3] Now look at the birds. Give a name for the number of birds. This name should tell what is happening. [1 + 4] Now look at the printed words. You are to complete this sentence. Which is correct, 1 + 4 is greater than 4 + 3 or 4 + 3 is greater than 1 + 4? Now write 4 + 3 on the first response space. Write 1 + 4 on the other response space."

Have the children complete the page. Warn them that they must be careful because some of the pictures are about "less than," some are about "equal to," and some are about "greater than."

STA 2 page 51

Sums and minuends through 8; ideas of "greater than," "less than," and "is equal to" [extension]

The children use phrases and standard names for numbers to continue their study of the ideas of "greater than," "less than," and "is equal to."

Three blocks of exercises are included on this page. Exercises A through Y (red letters) and exercises A through R (blue letters) contain most of the basic facts that the children have studied. Exercises A through U (brown letters) are concerned with the ideas of "greater than," "less than," and "equal to."

Direct attention to the first block of exercises. Have the children read exercise A silently. Ask them whether 8 - 3 is a phrase or the standard name for a number. They should say that 8 - 3 is a phrase. Next ask them to think of the standard name for the number named by 8 - 3. Then ask one child to read the exercise aloud, replacing the square screen with the correct numeral. Have other children verify or correct the reading of the completed exercise.

Use a similar procedure for each of exercises B through Y.

The exercises in the second block are similar to exercises A through Y except that printed words are used. Have the children read each exercise silently and decide on the standard name for the number that is expressed by a phrase. Then ask a child to read the exercise aloud, replacing the placeholder with the correct numeral. The other children should verify or correct the reading of the completed sentence.

Direct attention to the third block of exercises. Ask various children to read aloud each of the groups of words printed above the exercises. Then have the children read exercise A silently and decide which group of words should replace the wavy line in the exercise. Finally, have one child read the completed sentence aloud and have the other children verify or correct the reading of the completed sentence.

Practice tablet page 37
Workbook edition page 74

Call attention to Exercise A. Say: "What two numbers are named in this exercise? Is 2 + 3 less than 5 + 3? Is 2 + 3 greater than 5 + 3? Since 2 + 3 is less than 5 + 3, draw a ring around the words *is less than*. Now the sentence tells the truth about the numbers 2 + 3 and 5 + 3. Complete each of the other sentences. Remember to draw a ring around the words that belong in the sentence."

STA 2 page 52
Workbook edition page 75

Relationship between addition and subtraction [extension]

The children learn that subtractive action expressed by phrases such as 5 - 2 can be viewed as the "undoing" of additive action expressed by phrases such as 3 + 2.

Each of the pictures on this page consists of three scenes: the first scene pictures an additive situation; the second scene shows the total group; and the third scene pictures a subtractive situation. For each picture, the subtractive situation in the third scene pictures the "undoing" of the "joining" shown in the first scene. In each first and third scene, the set of moving objects is dimmed to focus attention on the stationary objects.

Direct attention to scene 1 of picture A. Ask the children to describe what they see. Have the children tell how many squirrels are not moving, how many squirrels are moving, and what numeral completes the phrase 1 + ▩ . For the second scene, ask questions to help the children understand that the set pictured shows the result of the "joining" action shown in the first scene. They should identify the number of squirrels in the second scene as 3. For the third scene, have them decide how many squirrels were in the original set, how many squirrels are running away, and what numeral completes the phrase 3 − ▩ . They should understand that the result of the action shown in the third scene tells how many squirrels remain. The number associated with this set is 1. Now ask questions to help the children understand that the number of squirrels originally in the first scene and the number of squirrels that remain in the third scene is the same. Then discuss how the action in the third scene "undoes" the the action shown in the first scene.

Discuss pictures B, C, and D in a similar manner.

STA 2 page 53

Relationship between addition and subtraction [extension]

The children continue to study the relationship between addition and subtraction. Use the pictures in alphabetical order. For each picture that shows additive action, have the children give the phrase that corresponds to the action (1 + 3 for picture A) and then find a picture that shows the "undoing" of this action and give its phrase (picture I, 4 − 3, for picture A).

Let the children discover that there is no picture showing subtractive action that is related to the joining action shown in picture E and there is no related picture showing additive action for picture L.

Do not hurry the work on this page. Have the children use markers at their desks to strengthen their understanding of the ideas presented on this page.

Practice tablet page 38

Workbook edition page 76

Call attention to the first part of Picture A. Say: "Give a name for the number of animals. This name should tell what is happening. [2 + 1] Find the response space that is in the first part of the picture. Write 2 + 1 on this response space. What is the standard name for 2 + 1? Now look at the second part of the picture. How many animals are there? Are there the same number of animals in each part of the picture? In this part, you are to show the action that undoes 2 + 1. How many animals should you take away? [1] Draw a ring around 1 animal. Give a name for the number of animals that will be left. [3 − 1] Find the response space that is in the second part of the picture. Write 3 − 1 on this response space."

Have the children complete the page. Be sure they understand that, for the first part of a picture, they are to write the name for the number of objects. For the second part of a picture, they first show the undoing action by drawing a ring around the objects they should take away and then they write the name for the number of objects that will be left.

STA 2 page 54

Checking up: ideas of "less than," "greater than," "equal to," betweenness; addition and subtraction basic facts

The children are tested on their understanding of the ideas of "less than," "greater than," "equal to," and betweenness for numbers and also on their knowledge of addition and subtraction basic facts through sums and minuends of 8.

You could use the tests on page 54 over a period of several days. The first test, exercises A through H (blue letters), tests understanding of the ideas of "one greater than" and "one less than." The second test, exercises A through H (brown letters), tests understanding of the idea of betweenness for numbers. The third test, exercises A through Y (red letters) and exercises A through Y (green letters), tests understanding of all the addition and subtraction basic facts that the children have studied up to this point. The fourth test, exercises A through L (blue letters), tests understanding of the ideas of "greater than," "less than," and "equal to."

The responses for each test may be given either orally or written. The teaching suggestions given for *STA 2* page 35 may be adapted to this page.

Practice tablet page 39
Workbook edition page 77

Notice that this page consists of two blocks of exercises. Use your own judgment as to how many exercises you can assign at one time.

Tell the children that they are to complete each of the exercises by writing the correct numeral on the response space.

Practice tablet page 40
Workbook edition page 78

Call attention to Picture A. Say: "Point W is the endpoint of one segment that is shown in this picture. Name the other endpoint of this segment. [Point R] Remember that you use the names of the two endpoints to name a segment. Name the segment that has point W and point R as endpoints. [The children can give either WR or RW as the name of this segment.] Find the response spaces that are under the word *segments*. Write WR (or RW) on the first response space. Is another segment shown in Picture A? [Yes] Name the endpoints of this segment. [Point R and point X] Name this segment. [The children can give either RX or XR as a name.] Write RX (or XR) on the next response space. Is there another segment shown? [No]

"For each of the other pictures, you are to name each segment that is shown. Then you write the two letters that name the segment on one of the response spaces. Study each picture carefully because sometimes only 1 segment is shown and sometimes 2 or more segments are shown."

PROBLEM SOLVING

An important goal of a modern arithmetic program is to provide the child with the concepts and techniques that will enable him to solve problems involving quantitative ideas. In the *Seeing Through Arithmetic* program, work on set recognition, on one-to-one correspondence, on recognition of the action in problem situations, on basic facts, and on building the mathematical sentence that describes a given situation provides a sound foundation upon which to build the understanding and skills needed for effective problem solving.

In *Book 2* of the *ST4* program, insight into problem solving is fostered by the systematic use of a distinctive teaching method. First,

the children are taught to recognize the action (what is happening) in a problem situation. Second, they learn to express the situation in mathematical symbolism. This symbolism includes the use of mathematical sentences (for example, 2 + 3 = ▮▮▮). Third, the children learn to perform the computation that is necessary to find a numerical replacement for the placeholder. The number named by this numerical replacement answers the question asked in the problem.

The difficulty that many children have with problems comes about because of their inability to analyze problem situations. The skills and techniques that underlie problem analysis must be taught from the earliest grades. At the onset, the child must learn to recognize the numerousness of sets. Then he must learn to identify the type of action that is pictured or is described in words in the problem. For example, in some situations one set is combined with another set, and the action indicated is *additive*. In other situations, some members of a set are removed from a given set, and the action indicated is *subtractive*. *Multiplicative* action and *divisive* action occur in still other situations. Children who learn to recognize each of these types of action are well on the way to developing a mathematical insight that is essential to problem solving.

Children learn to solve problems by working with "story" problems. A "story" problem is a verbal statement (or series of statements) that describes a physical situation and poses a question that requires a solution involving the use of quantitative data. In the *Seeing Through Arithmetic* program, the children first learn that a verbal problem is a description of something that has been done, is being done, or could be done. In other words, it is a description of a physical situation. Then they learn to visualize, that is, to see or imagine, the problem situation. At the very beginning, sets of objects shown in picture sequences help the children to visualize number situations and the actions performed by (or upon) the sets. The children should gradually become independent of such learning aids as pictures and objects in thinking about the problem situations, and, eventually, the actions of joining and separating should require only mental interpretation.

After the children can visualize the action in problem situations, they learn to use words and other symbols to say or write what is happening. They use numerals to tell how many objects are in a set (or sets) and other symbols (+, ×, −, ÷) to express the action.

You may ask, "How do children learn to associate meanings and symbols?" The answer to this question is complex, but the essential idea can be brought out by considering a nonquantitative experience. The symbol "cat" or the spoken word "cat" will be meaningful to a child only to the extent to which he has had experience with cats. The child who has pulled a cat's tail, who has been scratched by a cat, or who has had a cat for a pet can recall these experiences when he hears the spoken word "cat" or sees the symbol "cat." In other words, the symbol "cat" will, in the end, be associated with these experiences and thus have meaning for the child.

It seems clear that the elements of meaning are based upon experience--upon direct contact with objects through the senses of touch, sight, hearing, and smell. Number meanings are no exception. If the children have had a variety of experiences with sets of 6 objects and if they associate these experiences with the symbol 6, they are reacting meaningfully to the symbol 6. These experiences should include such activities as grouping 4 objects with 2 objects or 3 objects with 3 objects to make a set of 6, regrouping 6 objects into a set of 2 objects and a set of 4 objects, and experiencing all other possible groupings of 6 objects. Some of these experiences will naturally include ideas that are the foundations

for the operations of addition, subtraction, multiplication, and division.

In much the same way, such symbols as "+" and "−" become meaningful. The children formulate the generalization that the word "plus" and the symbol "+" are associated with those actions that result in the combining of two sets, as when a set of 4 balls is put with a set of 2 balls. Likewise, they formulate the generalization that the symbol "−" and the word "minus" are associated with those actions that result in the separating of some objects from a set of objects, as when a set of 5 apples is removed from a set of 6 apples. The effect of such a program of instruction on problem solving is evident. The children decide whether to add or subtract on the basis of what the physical situation suggests that they do with the sets involved rather than on the basis of "cue" words. Problem-solving procedures become meaningful to children because the symbols they use are related to the action implied in the problem situation.

It is convenient in arithmetic instruction to have some simple symbolic device for showing children that they are to find a particular number. In the *Seeing Through Arithmetic* program, the children learn first to recognize the "square screen" (■) or the "round screen" (●) as a placeholder in a mathematical sentence. They read ■ and ● by saying "what number." Now, as they learn to analyze problem situations, they use one of these symbols as a placeholder for the number that is wanted in a particular problem. For example, the sentence "3 + 4 = ■" shows that a set of 4 objects is joining (or has joined) a set of 3 objects, and the standard name for the number expressed as 3 + 4 is to replace ■. Remember that the phrase 3 + 4 describes an action situation and also names a number. The symbol for equality can be used in the sentence 3 + 4 = ■ because the replacement for the square screen is the standard name for the number named by 3 + 4. Consequently, we can say that three plus four is equal to seven.

Following the development of the mathematical sentence that expresses the problem situation, the children learn to use the appropriate computation that will produce the "answer" to the problem. In the simplest situations, this involves only the direct interpretation of the operation as indicated by the sentence. Sentences of the forms 3 + 2 = ■ and 5 − 1 = ■ are examples of this type. However, the importance of the distinction between the use of symbols to show what is happening and the processing of the symbols is seen when one recognizes that the process used is not always what the action seems to suggest. Verbal problems that involve sentences of the forms ■ + 2 = 5 and ■ − 3 = 1 are examples of this type.

Throughout the *Seeing Through Arithmetic* program, the basic types of problems introduced in each grade are selected from a complete list of such types. This list is the result of an analysis by the authors. It provides a guide for a consistent and comprehensive treatment of problem solving. In *Book 2* of the *STA* program, the children are introduced to verbal problems that involve sentences of the following forms: 3 + 2 = ■, 6 − 4 = ■, ■ + 3 = 5, and ■ − 3 = 2. In this *Teaching Guide*, each of these basic types of problems is dealt with in the lesson notes. Each type is presented by a systematic teaching development that leads to understanding.

Throughout this work on problem solving, you can reinforce the experiences provided in the book by allowing the children to use objects and markers at their desks. Using these objects and markers, the children should show the actions that are pictured or described in words. That is, to show a set of 3 objects joining a set of 4 objects, the child should place 4 objects on his desk. Then he should assemble a set of 3 objects and push this set over to join the set of 4. To show a set of

5 being removed from a set of 8, he should place a set of 8 objects on his desk and then remove a set of 5. Any kind of small object is suitable for such use (for example, bottle caps, corks, jacks, pieces of cardboard, and so on). Avoid objects that have sharp edges, or that roll easily, or that are so light they might be blown off the desks.

STA 2 page 55
Workbook edition page 79

Problem solving: situations involving additive action -- sentences of the form 3 + 2 = ▮

The children have learned to recognize the numerousness of sets of objects. They can recognize additive action and give a phrase that describes the problem situation and tells the number of objects in the combined sets. They know the standard name that can be used for this number. Now they begin to learn to apply these ideas to solving "story" problems. They use pictures to interpret a verbal problem that involves additive action in which a set of objects joins (or has joined) another set of objects. The number of objects in each set is known. The children express the problem situation by making a mathematical sentence, and they find the number that answers the question asked in the problem.

In teaching this page, first have the children look at the page as a whole. Then direct attention to the pictures and ask the children to describe what they see. They will probably discover that the pictures "tell a story." Have them put their markers on their desks. Tell them to imagine that the markers are pigs and ask how they can use the markers to tell the story about the pigs. Let each child place a set of 3 markers on his desk. He should next place a set of 2 markers on the desk near the set of 3 markers. Then he should push the set of 2 markers over to join the set of 3 markers to make a set of 5. Direct attention to the sentences beside the pictures and explain that these sentences also tell the story about the pigs. Have the children read the sentence beside picture A. Be sure they understand that both picture A and the sentence describe the same situation, namely, 3 pigs are eating. Then direct attention to the numeral 3 at the right. Get the children to decide that this numeral expresses the number associated with the set of pigs shown in picture A. Draw attention to the numeral 3 in the sentence "3 pigs are eating."

Refer to picture B. Point out the set of 3 pigs that is dimmed. Explain that this is the set that is shown in picture A, but it is now dimmed, so we will pay attention to something else in the picture. Ask the children to describe what is happening in picture B. They should say that 2 more pigs are coming to eat. Ask: "Does the sentence at the right describe what is happening in picture B? What word can you use to describe the action when one set of objects is joining another set of objects?" The children should decide that the word "plus" is the appropriate word. Then direct attention to the phrase 3 + 2 and ask the children to explain why the numeral 3 is dimmed. They should say that this is done so we will pay attention to the symbols that show what is happening in picture B. Get the children to explain what the symbol "+" shows (the action is a joining action) and what the numeral 2 expresses (the number associated with the set of pigs that are coming to eat).

Direct attention to picture C. Ask what sets are shown in this picture. Have the sentence beside picture C read. Then direct attention to the phrase 3 + 2 and the mathematical sentence 3 + 2 = ▮ at the right. Ask: "What does the phrase 3 + 2 tell you? (That a set of 2 objects has joined a set of 3 objects) What does the sentence 3 + 2 = ▮ tell you? (That they are to find the standard name for the number 3 + 2)" Then explain that they should read the sentence 3 + 2 = ▮ as "Three plus two is equal to what number?"

Remind the children that the phrase 3 + 2 names a number. They use the symbol for "is equal to" because they need to find the standard name for the number 3 + 2.

For picture D, the children should observe that 5 pigs are eating and should read the sentence beside the picture. Then ask them to read silently the mathematical sentence 3 + 2 = 5. Have various children read the sentence aloud to ensure that they understand that it is read "Three plus two is equal to five." Ask: "What is the standard name for 3 + 2?" Emphasize that in this case we replace the square screen in the mathematical sentence with the standard name. Then ask a child to read aloud the question asked in the problem (picture C). Ask: "Does 5 answer the question asked in the problem?" The children should say that 5 answers the question asked in the problem.

STA 2 page 56
Workbook edition page 80

Problem solving: situations involving additive action -- sentences of the form 3 + 2 = ■

The children continue to analyze verbal problems that involve additive action, in which a set of objects joins (or has joined) another set of objects. The number of objects in each set is known. They learn how to build the mathematical sentence that expresses the problem situation, and they use this sentence to find the number that answers the question asked in the problem.

On this page, problem A and pictures B, C and D form one teaching sequence. Problem E and pictures F, G, and H form another sequence. Note that a problem is given in its usual verbal form at the beginning of each sequence. The pictures below each verbal problem illustrate the original situation described in the problem, the action, and the result. The work on this page again emphasizes the idea that a verbal problem describes a physical problem situation and poses a question that requires a numerical solution. Also emphasized is the development of a mathematical sentence that expresses the original situation, the action, and the result.

Direct attention to the problem labeled A and tell the children to read it silently. Have them imagine that their markers are puppies, and ask them to show with their markers what is happening to the two sets of puppies. The children should put a set of 4 markers on their desks and place another set of 2 markers near the set of 4 markers. Then they should push the set of 2 markers over to join the set of 4 markers. Avoid asking for the number associated with the combined sets at this time.

Direct attention to picture B. Have the children describe what they see. Then have one child read aloud the sentence in the picture. Ask what the numeral 4 in picture B expresses (the number of puppies that are playing). It is important that the children understand that the picture, the sentence, and the numeral 4 all describe the problem situation as it was at first, before any action took place.

Direct attention to picture C, and ask for a description of what is taking place. The children should observe that the set of 4 puppies is dimmed to focus attention on the puppies that are moving (the action pictured). They should say that 2 puppies are joining the 4 puppies. Have the children read the sentence in picture C. Then ask what word can be used for the action shown in the picture and described in the sentence "Two other puppies are running to play." The children should decide that "plus" is the appropriate word. Next, discuss the phrase given below the sentence, so that the children understand the meaning of each symbol: the dimmed numeral "4" expresses the number associated with the set of puppies that are playing; the symbol "+" describes the joining action; and the "2" names the number associated with the set of puppies that is joining the set of 4. Then ask a child to find

and read the question asked in the problem (picture D). Have another child find and read the mathematical sentence. He should read 4 + 2 = ▮▮ in picture D as "Four plus two is equal to what number?" Discuss the sentence to ensure that the children understand that the sentence serves two purposes: first, it expresses the situation described in the problem; and, second, it indicates that the standard name for the number of the combined sets is wanted.

Ask: "What is the next thing we should do? [The children should explain that they must find the standard name for the number 4 + 2.] What is the standard name for 4 + 2?" Then ask one child to read aloud the question asked in the verbal problem. Ask another child to answer the question. Be sure that the children understand that the sentence 4 + 2 = ▮▮ describes the problem, that 6 is the standard name for the number 4 + 2, and that the sentence "6 puppies are playing" answers the question asked in the problem.

Discuss problem E, illustrated in pictures F, G, and H, in a similar manner. The children should understand that a set of 1 is joining a set of 3 and should develop the mathematical sentence (3 + 1 = ▮▮) that describes the problem. They should then decide that 4 is the standard name for 3 + 1 and that the sentence "4 kittens are in the yard now" answers the question asked in the problem.

Some of the children will profit from further use of markers.

STA 2 page 57
Workbook edition page 81

Problem solving: situations involving additive action -- sentences of the form 3 + 2 = ▮▮

The children continue to analyze verbal problems that involve additive action and to develop mathematical sentences for these problem situations.

Problem A and pictures B, C, and D form one teaching sequence. Problem E and pictures F, G, and H form another teaching sequence.

Direct attention to problem A and have the children read it silently. Ask them to imagine what is happening in the problem and have them demonstrate the action with their markers.

Direct attention to picture B and ask the children to describe what they see. Ask what the numeral 4 names (the number associated with the set of cars on the table).

Discuss picture C. Ask what is happening (a set of 1 is joining the set of 4). Ask the children to tell the meaning of each numeral and symbol used in the phrase 4 + 1. They should decide that the numeral 4 expresses the number associated with the set of cars that are on the table, that the symbol "+" expresses a joining action, and that the numeral 1 is associated with the set of cars that Jim put on the table.

For picture D, the children should observe that the number wanted is the number of the combined sets (or the standard name for 4 + 1). They should decide that 4 + 1 = 5 and that 5 is the number associated with the combined sets. Ask one child to read problem A aloud. Ask another child to give the mathematical sentence that describes the problem situation. [Four plus one is equal to what number?] Ask another child to give the standard name for 4 + 1 and the sentence that answers the question asked in the problem. [5 toy cars were on the table then.]

Discuss problem E and pictures F, G, and H in a similar manner. The children should understand that a set of 4 is joining a set of 3, and they should work through the pictures to build the sentence 3 + 4 = ▮▮, which expresses the problem situation. Then they should decide that 3 + 4 = 7 and that 7 is the number associated with the combined sets. Encourage a child to explain that the sentence

"7 chickens were eating" answers the question asked in the problem.

Children who have trouble with either of the problems on this page should use markers or other objects to show the action and to find the replacement for the screen.

STA 2 page 58
Workbook edition page 82

Problem solving: situations involving additive action -- sentences of the form 3 + 2 = ■

The children continue to analyze verbal problems that involve additive action. They develop the mathematical sentence that corresponds to the problem situation and use the sentence to find the number that answers the question asked in the problem.

On this page, four verbal problems are presented. A single picture illustrates the action for each problem. Below each problem is given the mathematical sentence that describes the problem situation. (For example, "5 + 3 = ■ " is given for the first problem.)

Have the children silently read the verbal problem beside picture A. Ask how picture A helps them to imagine what is happening in the problem. The children should explain that the picture shows 3 ducks joining 5 ducks. Say: "Show this action with your markers. Now use your cards with numerals and other symbols on them to build the phrase that describes the action." (Each child should build the phrase 5 + 3.) Then call attention to the question asked in the problem. Have the children complete the sentence that expresses the situation in the problem and tell what must be done to obtain the answer to the question. The children should complete 5 + 3 to read 5 + 3 = ■. Have the children compare their sentences with the sentence shown below the problem in A. Ask: "Is it the same as the sentence you made?" Ask the children to interpret the sentence 5 + 3 = ■ in terms of the problem and the picture. The children should decide that 5 is the number associated with the set of ducks that were in the pond, that "+" expresses a joining action, and that 3 is the number associated with the set of ducks that came to the pond. The children should understand that the square screen indicates that the number associated with the combined sets is to be found, and they should decide that this number is 8. Ask: "Why can 'is equal to' be used in the sentence?" [Because 5 + 3 and 8 are names for the same number]

Have the children reread the problem. Ask: "Is 8 the number of ducks that were in the pond? Give the answer to the question asked in the problem." [Then 8 ducks were in the pond.]

Use a similar procedure for each of the remaining pictures and problems. As a final activity, you may tell the children to reread each problem and write the mathematical sentence that describes the situation, the action, and the result.

Children who have difficulty with the work on this page should be given special help. Write several problems similar to those on this page on the chalkboard. Provide the children with markers. For each problem on the board, have them show the action with their markers. After they have done this, have them use their cards to build the mathematical sentence that describes the problem situation. Then have them give orally (or write) the completed mathematical sentence, supplying the correct replacement for the square screen, and give the sentence that answers the question asked in the problem.

STA 2 page 59

Problem solving: situations involving additive action -- sentences of the form 3 + 2 = ■

The children continue to analyze verbal problems in which a set of objects joins

another set of objects. They build the mathematical sentence for each problem and find the number that answers the question asked in the problem.

The teaching procedures given for using *STA 2* page 58 may be adapted to this page.

Some of the children may be able to do the work on this page independently. After they have built the sentences with their cards, have them write two sentences for each problem. The first sentence should contain a placeholder (▮) to indicate the number that is to be found. The second should contain the numeral that replaces the placeholder.

Work with the slower children in the way suggested in the notes for *STA 2* page 58. Give them enough time so that they can work with assurance. Do not leave the page until each child can build and write a mathematical sentence for each problem on the page. Permit use of objects and markers.

Practice tablet page 41
Workbook edition page 83

Call attention to Picture A and its problem. Say: "Read the problem silently. Now look at the picture. What is happening? Do the picture and the problem tell the same story? What are you to find for this problem? [How many toy rabbits in all were on the table] To find this number, you make the arithmetic sentence that describes the problem. How should you begin your sentence? [By using 1 + 1 to show that 1 rabbit was put with 1 other rabbit] What should you use next? [The symbol for *is equal to*] What should you use now? [A screen] Why do you use a screen? [To hold the place for the number you are to find] Write 1 + 1 = ▮ on the first response space. Now you must find the numeral that belongs where the screen is. What is this number? [2] On the second response space, write 1 + 1 = 2.

"For each of the other problems, read the problem and study the picture. Next, on the first response space, write the sentence that describes the problem. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is."

Practice tablet page 42
Workbook edition page 84

Say: "Read each problem carefully and study the picture that is beside it. Next, on the first response space, write the sentence that describes the problem. Be sure to use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is."

STA 2 page 60

Keeping skillful: situations involving additive action -- sentences of the form 3 + 2 = ▮ ; sums and minuends through 8

The children analyze and solve verbal problems that involve additive action in which one set of objects joins another set of objects. They also practice the addition and subtraction basic facts that they have studied so far.

Some of the children may be able to do the work on this page independently. Have them read each problem, write on their papers the letter that labels it, and write two sentences. The first sentence should express the problem situation and should contain a placeholder. (The placeholder can be a square screen, ▮, or a round screen, ◍, or simply a box, □. If the children use a box, caution them not to write numerals inside the box as in ⚀.) The second should contain the numeral that replaces the placeholder. As an example, for problem A,

they should write the following:

A. 2 + 3 = ■ 2 + 3 = 5

The other children should participate in activities similar to those you have been using for previous problem-solving pages. Each problem should be read silently first. Then the children should show the action with markers or objects and use their cards to build the mathematical sentence. After problems A through H have been worked through in this way, have each problem reread, and have the children find the sentence in the first block of exercises at the right (blue letters A through L) that shows what is happening in the problem. For problem A they should select sentence E (2 + 3 = ■) as the appropriate sentence. Ask a child to reread sentence E, supplying the correct replacement. Tell the children to think of a sentence that answers the question asked in the problem. Then ask various children to give their responses. They should say, "There were 5 turtles in the pond."

Both blocks of exercises at the right (blue letters A through L and red letters A through M) may be used for oral or written practice on the basic facts with sums and minuends through 8. Have the children take turns in reading the exercises. The children may respond orally, in turn, giving the sentence again with the correct numeral for the placeholder, or they may write the completed sentence on a sheet of paper opposite its labeling letter. If the responses are written, supply the correct responses for the children so they can verify their work.

Practice tablet page 43
Workbook edition page 85

Call attention to Problem A. Say: "Read the problem silently. Think about what is happening in this problem. What are you to find for this problem? [How many toy wagons Tom had in all] To find this number, you make the arithmetic sentence that describes the problem. How should you begin your sentence? [By using 1 + 2 to show that Tom had 1 toy wagon and was given 2 more] What should you use now? [The symbol for *is equal to*] What should you use now? [A screen] Why do you use a screen? [To hold the place for the number you are to find] Write 1 + 2 = ■ on the first response space. Now you must find the numeral that belongs where the screen is. What is this numeral? [3] On the second response space, write 1 + 2 = 3.

"For each of the other problems, read the problem. Think about what is happening in this problem. Next, on the first response space, write the sentence that describes the problem. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is."

Either before the children begin to work, or after they have completed Problems A through F, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

Practice tablet page 44
Workbook edition page 86

Say: "Read each problem. Think about what is happening in the problem. Next, on the first response space, write the sentence that describes the problem. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is."

Either before the children begin to work, or after they have completed Problems A through F, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

STA 2 page 61
Workbook edition page 87

Problem solving: situations involving subtractive action -- sentences of the form 6 - 4 = ■

The children now learn to interpret a verbal problem that involves subtractive action. In such a problem, some members of a set are removed or move away. The children express the problem situation by developing a mathematical sentence, and they find the number that answers the question asked in the problem.

First let the children look at pictures A through D. They will undoubtedly discover that the pictures "tell a story." Have the children look again at picture A and read the sentence beside the picture. Then direct attention to the numeral 6 at the right. Ask what this numeral names (the number associated with the set of dogs that are playing). Emphasize that the numeral 6 tells how many dogs there are to begin with.

Refer to picture B and ask what is happening. The children should observe that 4 of the dogs are running away. Call attention to the set of dogs that is dimmed and explain that these dogs will remain. Then, have them read the sentence that tells what is happening. Ask what mathematical word can be used for the action shown in the picture and told about in the sentence. The children should decide that the word "minus" is appropriate. Direct attention to the phrase at the right. Ask why the numeral 6 is dimmed (so we will pay attention to the symbols that describe the action). Be sure the children understand that the symbol "-" shows that there is a separating action and that the numeral 4 names the number associated with the set of dogs that are running away.

Tell the children to look at picture C. Ask how picture C differs from picture B. (Picture C shows all the action described in the problem. We can now visualize the situation about which we have information.) Say: "The sentence beside picture C asks a question. Read this sentence." Then direct attention to the phrase 6 - 4 and the sentence 6 - 4 = ■ at the right. Ask: "What does 6 - 4 tell you? What does 6 - 4 = ■ tell you? (The children should say that 6 - 4 tells that 4 objects are moving away from 6 objects; 6 - 4 = ■ tells that 4 objects are moving away from 6 objects and the number of objects in the set that remains (or the standard name for 6 - 4) is wanted. Remind the children that 6 - 4 names a number and describes the number in terms of the action. They can use the symbol "=" or the words "is equal to" because 6 - 4 and the replacement for the placeholder name the same number.

In picture D the children should observe that 2 dogs are playing. Have them read the sentence beside picture D (2 dogs are playing). Ask: "Does this sentence answer the question asked in the problem? Look at the sentence 6 - 4 = 2. Is 2 the standard name for 6 - 4? Can you use the number 2 to give the answer to the problem?" Have a child read the entire verbal problem aloud (the sentences given in A, B, and C). Have another child give the answer to the question asked. [2 dogs are playing.]

As a variation on the procedures just outlined, or as an adaptation for a special group of children, you may wish to have the children show the action portrayed in pictures A through D by using objects or markers on their desks before you start work on the verbal problem itself.

STA 2 page 62
Workbook edition page 88

Problem solving: situations involving subtractive action -- sentences of the form 6 - 4 = ■

The children continue to analyze verbal problems that involve subtractive action in which some members of a set move away or are

removed. They learn to build the mathematical sentence that expresses the problem situation and to use the sentence to find the number that answers the question asked in the problem.

Problem A and pictures B, C, and D form one teaching sequence on this page. Problem E and pictures F, G, and H form another sequence. Note that a problem in its usual verbal form is given at the beginning of each sequence. The three pictures below each verbal problem illustrate the original situation described in the problem, the action, and the result. The work on this page emphasizes the development of a mathematical sentence that "fits" a problem involving subtractive action.

Direct attention to the problem labeled A and have the children read it silently. Tell them to imagine that their markers are chickens and to show with their markers what is happening to the set of 5 chickens. The children should put 5 markers on their desks and then remove 3 markers. Avoid asking at this time for the number associated with the remaining set.

Direct attention to picture B. Have the children describe what they see and have one child read aloud the sentence in the picture. Ask what the numeral 5 in picture B expresses (the number associated with the chickens that are eating). The children should realize that the picture, the sentence, and the numeral all describe the problem situation as it was at first, before any action took place.

Ask what is happening in picture C. The children should observe that 3 chickens are leaving, and that the picture of the remaining chickens is dimmed because we are interested at the moment in the chickens that are going away. Have the children read the sentence in the picture. Then ask what word can be used for the action shown in the picture and described in the sentence. The children should decide that "minus" is the appropriate word. Say: "Look at the phrase 5 - 3. What does the 5 tell you? What does the minus sign tell you? What does the numeral 3 tell you?" The children should decide that the numeral 5 expresses the number associated with the chickens that were eating, that the symbol "-" shows there is a separating action, and that the numeral 3 expresses the number associated with the chickens that are leaving.

Then ask a child to find and read the question asked in the problem (picture D). Ask another child to read the mathematical sentence that describes what is happening in the problem and also tells that the number of the remaining set is wanted. He should read 5 - 3 = ■ in picture D as "Five minus three is equal to what number?"

Now ask: "What should we do next?" The children should explain that they must find the standard name for 5 - 3. They should look at picture D, see that there are 2 chickens remaining and decide that 5 - 3 = 2. Ask a child to reread the verbal problem. Ask another child to give the sentence that describes the problem situation ("Five minus three is equal to what number?") and then give the sentence again, this time supplying the correct replacement. Have a third child give the answer to the question asked. [2 chickens were eating then.]

Use similar procedures with problem E and pictures F, G, and H. The children should understand that 1 cat is leaving the original set of 6 cats and should develop the mathematical sentence that describes the problem situation (6 - 1 = ■). Then they should decide that 5 is the standard name for 6 - 1 and should give the answer to the question asked in the problem. [Then 5 kittens were sleeping.]

STA 2 page 63
Workbook edition page 89

Problem solving: situations involving subtractive action -- sentences of the form 6 - 4 = ■

The children continue to analyze verbal problems that involve subtractive action. They develop the mathematical sentence that expresses the problem situation and use the sentence to find the number that answers the question asked in the problem.

Problem A and pictures B, C, and D form one teaching sequence. Problem E and pictures F, G, and H form another teaching sequence.

Have the children look at the sequence of pictures on the left side of the page as a whole. Then direct attention to problem A and have the children read it silently. Ask them to imagine what is happening in the problem and have them demonstrate the action with their markers.

Direct attention to picture B and ask the children to describe what they see. Ask what the numeral 3 names (the number associated with the set of ponies in the field).

Discuss picture C. Ask: "What is happening? Look at the phrase 3 - 2. What does the 3 tell you? What does the minus sign tell you? What does the 2 tell you?" The children should decide that the numeral 3 expresses the number associated with the original set, that the symbol "-" shows there is a separating action, and that the numeral 2 is associated with the set of ponies that is leaving.

For picture D, the children should observe that the number to be found is the number of ponies in the remaining set (the standard name for 3 - 2). They should decide that 3 - 2 = 1 and that 1 is the number associated with the remaining set. Ask one child to read problem A aloud. Ask another child to give the mathematical sentence that describes the problem situation. ("Three minus 2 is equal to what number?") Ask a third child to read the sentence again, this time using the correct replacement for the square screen (3 - 2 = 1) and to give the answer to the question asked in the problem. [Then 1 pony was in the field.]

Analyze problem E and its accompanying pictures in a similar way. The children should understand that 3 jacks are removed from the 8 jacks and should work through the pictures to develop the sentence that describes the problem situation (8 - 3 = ■). Then they should decide that 8 - 3 = 5, that 5 is the number associated with the remaining set, and that the sentence "Nancy has 5 jacks left" answers the question asked in the problem.

Children who have trouble with either of the problems on this page should use markers to show the action and to find the replacement for the screen.

STA 2 page 64
Workbook edition page 90

Problem solving: situations involving subtractive action -- sentences of the form 6 - 4 = ■

The children continue to analyze verbal problems that involve subtractive action. They develop the mathematical sentence that expresses the problem situation and use the sentence to find the number that answers the question asked in the problem.

On this page, a single picture illustrates the action for each verbal problem. Below each problem is given the mathematical sentence that expresses the problem situation. (For example, 4 - 1 = ■ is given for the problem in picture A.)

Have the children silently read the verbal problem beside picture A. Ask how the picture helps them to imagine what is happening in the problem. The children should explain that 1 picture is shown being removed from the set of 4 pictures. Say: "Show this action with your markers." (Each child should put 4 objects on his desk and then remove 1 of the objects.) Say: "Now use your cards with numerals and other symbols on them to build the phrase that

describes the action." (Each child should build the phrase 4 − 1.) Then ask: "What question is asked in the problem? What should you do now to get the sentence that will help you to solve the problem?" (The children should complete 4 − 1 to read 4 − 1 = ■). Say: "You have made a mathematical sentence to help you answer the question asked in the problem. Look at the sentence that is below the problem in your book. Is it the same as the sentence you made? What does 4 tell you? What does the minus sign tell you? What does the 1 tell you? What does the placeholder stand for? Why is 'is equal to' used?" The children should decide that 4 is the number of pictures that Jane had, the symbol "−" is used to show the separating action, 1 is the number of pictures that Jane gave away, the placeholder stands for the number of pictures that Jane had left, and "=" is used because the replacement for ■ and 4 − 1 are names for the same number. Then say: "What is the standard name for 4 − 1? What numeral replaces the placeholder in 4 − 1 = ■ ? How many pictures did Jane have left? Read the problem again. Have we answered the question asked in the problem?"

Use a similar procedure for each of the remaining problems. As a final activity, you can have the children reread each problem and write the appropriate mathematical sentence.

Children who have difficulty with the work on this page should be given special help. Write on the chalkboard several problems similar to those on this page. Provide the children with markers and, for each problem on the board, have them show the action with their markers. After they have done this, have them use their cards to build appropriate mathematical sentences. Then have them give orally (or write) the completed mathematical sentence, supplying the correct replacement for the square screen.

STA 2 page 65

Problem solving: situations involving subtractive action -- sentences of the form 6 − 4 = ■

The children continue to analyze verbal problems that involve subtractive action. They build a mathematical sentence for each problem and use the sentence to find the number that answers the question asked in the problem.

The teaching procedures given for using *STA 2* page 64 may be adapted to this page.

Some of the children may be able to do the work on this page independently. After they have built the sentences with their cards, have them write two sentences for each problem. The first sentence should express the problem situation and should contain a placeholder. The second should contain the numeral that replaces the placeholder. For problem A, they should write:

A 6 − 3 = ■ 6 − 3 = 3

Work with the slower children in the way suggested for *STA 2* page 64. Give the children enough time, so they can work with assurance. Do not leave the page until each child can build the mathematical sentence for each problem on the page. Permit use of objects or markers.

Practice tablet page 45

Workbook edition page 91

Call attention to Picture A and its problem. Say: "Read the problem silently. Now look at the picture. What is happening? Do the picture and the problem tell the same story? What are you to find for this problem? [How many toys were left on the table] To find this number, you make the arithmetic sentence that describes the problem. How should you begin your sentence? [By using 3 − 1 to show that 3 toys were on the table and 1 fell off the table] What should you use next? [The symbol for *is equal to*] What should you use now? [A screen] Why do you use a screen?

[To hold the place for the number you are to find] Write 3 − 1 = ▮ on the first response space. Now you must find the numeral that belongs where the screen is. What is this numeral? [2] On the second response space, write 3 − 1 = 2.

"For each of the other problems, read the problem and study the picture. Next, make the sentence that describes the problem and write it on the first response space. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is."

Practice tablet page 46
Workbook edition page 92

Say: "Read each problem and study the picture that is beside it. Next, make a sentence that describes the problem and write it on the first response space. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is."

STA 2 page 66

Keeping skillful: situations involving subtractive action -- sentences of the form 6 − 4 = ▮; addition and subtraction basic facts

The children analyze and solve verbal problems that involve subtractive action. They reinforce the learning of the addition and subtraction basic facts that they have studied so far.

Some of the children may be able to do the work on this page independently. Have them read each problem and write the labeling letter on their papers. Then, for each problem, they should write two sentences. The first sentence expresses the problem situation and contains a placeholder. The second sentence contains the numeral that replaces the placeholder. For problem A, they should write the following:

A 2 − 1 = ▮ 2 − 1 = 1

The other children in the class should participate in activities similar to those you have been using for previous problem-solving pages. Each problem should be read silently first. Then the children should show the action with markers or objects and use their cards to build the mathematical sentence that shows what is happening. After problems A through H have been worked through in this way, have each problem reread, and have the children find the sentence in the first block of exercises at the right (blue letters A through L) that shows what is happening in the problem. For problem A they should select sentence E (2 − 1 = ▮) as the appropriate sentence. Ask a child to reread sentence E, supplying the correct replacement, and then to give the answer to the problem.

Both blocks of exercises at the right (blue letters A through L and red letters A through M) may be used for oral or written practice on the basic facts with sums and minuends through 8. Have the children take turns in reading the exercises. The children may respond orally, in turn, giving the sentence again with the correct numeral for the placeholder, or they may write the completed sentence on a sheet of paper opposite its labeling letter. If the responses are written, supply the correct responses so that the children may verify their work.

Practice tablet page 47
Workbook edition page 93

Call attention to Problem A. Say: "Read the problem silently. Think about what is happening in this problem. What are you to find for this problem? [How many balls Nancy

had left] To find this number, you make the arithmetic sentence that describes the problem. How should you begin your sentence? [By using 4 - 1 to show that Nancy had 4 balls and then lost 1 of them] What should you use now? [The symbol for *is equal to*] What should you use now? [A screen] Why do you use a screen? [To hold the place for the number you are to find] Write 4 - 1 = ■ on the first response space. Now you must find the numeral that belongs where the screen is. What is this numeral? [3] On the second response space, write 4 - 1 = 3.

"For each of the other problems, read the problem. Next, make the sentence that describes the problem and write this sentence on the first response space. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is."

Either before the children begin to work, or after they have completed Problems A through F, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

Practice tablet page 48
Workbook edition page 94

Say: "Read each problem carefully and think about what is happening in the problem. Next, make the sentence that describes the problem and write this sentence on the first response space. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is."

Either before the children begin to work, or after they have completed Problems A through F, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

STA 2 page 67
Workbook edition page 95

Problem solving: situations involving additive action and subtractive action -- sentences of the forms 3 + 2 = ■ and 6 - 4 = ■

The children decide which kind of action is described in the verbal problem — additive action or subtractive action. They develop the mathematical sentence that describes the given situation.

The work on the page requires the children, for the first time, to distinguish between additive action and subtractive action as described in verbal problems. A picture accompanies each problem to help them make the distinction. A partially completed mathematical sentence also accompanies each problem. The wavy line (~~~) in each sentence is a placeholder for the symbol "+" or the symbol "-." The children must decide which of these symbols they should use to replace each wavy line.

Direct attention to the problem beside picture A and tell the children to read the problem silently. Have each child use his markers to show the action. Next direct attention to the partially completed sentence below the problem. Say: "We are going to complete this sentence. Think about the numerals 1 and 4 that you see in your book. What does the 1 tell you? What does the 4 tell you? What happened to the two sets of turtles? What symbol do we use to express a joining action? What symbol replaces the wavy line? Does 1 + 4 = ■ describe what happened?"

Have a child read the sentence in the book, replacing the wavy line with the word "plus." Have another child read it again, replacing both the wavy line and the placeholder by the appropriate symbols. Ask: "What is the stand-

ard name for 1 + 4? What number answers the question in the problem? How many turtles were in the box then?"

Work with the other three problems on this page in much the same way. Make suitable adaptations for the two problems that involve subtractive action. As a final activity, tell the children to read each problem silently and then, for each problem, write on paper the mathematical sentence that "fits" the problem.

Practice tablet page 49
Workbook edition page 96

Call attention to Picture A and its problem. Say: "Read the problem silently. Now look at the picture. What is happening? Do the picture and the problem tell the same story? Is this problem about a joining action or about a separating action? [A separating action] What are you to find for this problem? [How many jacks were left on the table] To find this number, you make the arithmetic sentence that describes the problem. Should you use the symbol for *plus* or the symbol for *minus* in your sentence? [The symbol for *minus*] How should you begin your sentence? [By using 6 − 3 to show that 6 jacks were on the table and 3 of the jacks fell off] What should you use next? [The symbol for *is equal to*] What should you use now? [A screen] Why do you use a screen? [To hold the place for the number you are to find] Write 6 − 3 = ▥ on the first response space. Now you must find the numeral that belongs where the screen is. What is this numeral? [3] On the second response space, write 6 − 3 = 3.

"For each of the other problems, read the problem and study the picture. Next, make the sentence that describes the problem and write this sentence on the first response space. Then, on the second response space, write the sentence with the numeral that belongs where the screen is. Be sure you read each problem carefully because some problems are about joining action and some are about separating action."

STA 2 page 68

Keeping skillful: situations involving additive action and subtractive action -- sentences of the forms 3 + 2 = ▥ and 6 − 4 = ▥ ; sums and minuends through 8

The children apply the skills and techniques they have been studying to the solution of problems that involve either additive action or subtractive action. They reinforce the learning of addition and subtraction basic facts that they have studied.

If possible, each child should work independently on this page. As a preliminary activity you may wish to go over the problems orally with the children. Then tell them to read each problem, write its label on their papers, and write two sentences for the problem. The first sentence should be of the form 4 + 5 = ▥ or the form 5 − 3 = ▥ , and the second should be of the form 4 + 5 = 9 or the form 5 − 3 = 2.

Use the two blocks of exercises at the right (orange letters A through L; brown letters A through M) for oral or written work.

Practice tablet page 50
Workbook edition page 97

Call attention to Problem A. Say: "Read the problem silently. Think about what is happening in this problem. Is this problem about a joining action or a separating action? [A joining action] What are you to find for this problem? [How many hats in all were on the table] To find this number, you make the arithmetic sentence that describes the problem. Should you use the symbol for *plus* or the symbol for *minus* in your sentence? [The symbol for *plus*] How should you begin your sentence? [By using 1 + 5 to show that 1 hat

was on the table and 5 more hats were put on the table] What should you use now? [The symbol for *is equal to*] What should you use now? [A screen] Why do you use a screen? [To hold the place for the number you are to find] Write 1 + 5 = ■ on the first response space. Now you must find the numeral that belongs where the screen is. What is this numeral? [6] On the second response space, write 1 + 5 = 6.

"For each of the other problems, read the problem carefully and think about what is happening in the problem. Next, make the sentence that describes the problem and write this sentence on the first response space. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is. Be sure you read each problem carefully because some problems are about joining action and other problems are about separating action."

Either before the children begin to work, or after they have completed Problems A through F, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

Practice tablet page 51
Workbook edition page 98

Say: "Read each problem carefully and think about what is happening in the problem. Next, make the sentence that describes the problem and write this sentence on the first response space. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is. Be sure you read each problem carefully because some problems are about joining action and other problems are about separating action."

Either before the children begin to work, or after they have completed Problems A through F, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

BASIC FACTS

As noted previously, the basic facts with sums and minuends of 2 through 8 were introduced in *Seeing Through Arithmetic 1*. These basic facts were limited to addition facts with addends of 1, 2, 3, and 4 and to subtraction facts with subtrahends and differences of 1, 2, 3, and 4. In the first presentation of basic facts on *STA 2* pages 28 through 47 (Wkbk pages 45 through 67), these facts were retaught and the groupings extended to include all basic facts with sums and minuends through 8.

Now the basic facts with sums and minuends of 9 and of 10 are presented in a similar manner.

STA 2 page 69
Workbook edition page 99

Basic facts: sums and minuends of 9

The children first learned to give a mathematical phrase, such as 3 + 2, that names a number and describes the number in terms of the action. Then they learned what is meant by standard names for numbers. Now they learn the standard name for phrases that express additive situations in which the resulting set has 9 objects. They also learn the standard names for phrases that express subtractive situations in which the original set has 9 objects (9 is the minuend).

Each picture on this page has two scenes. The first scene shows the action that is taking place; the second scene shows the set

that results from this action. You may wish to provide each child with a set of 9 markers that he can use as each picture is discussed.

Direct attention to the first scene in picture A. Have the children describe what is happening. Then ask them to give the phrase (8 + 1) that expresses this additive situation and tells how many objects there are. Have the children identify the number of objects (9) in the set shown in the second scene. They should recognize the numerousness of the set by grouping the objects, not by counting. Ask if the number of balls shown in the second scene is the same as the number of balls in the first scene. Encourage them to think of 8 + 1 and 9 as names for the same number. Then they should decide that, since 8 + 1 and 9 are names for the same number, they can say "8 + 1 = 9." They should identify 9 as the standard name for 8 + 1. Finally have a child read aloud the sentence given in picture A.

Follow a similar procedure for picture B, and, later, for E and F.

For picture C, direct attention to the first scene. Ask how many cans were in the original set. Have the children tell what is happening. Then ask them to give the phrase (9 − 1) that expresses the action and also tells how many cans are left. Encourage them to discover that the set of cans shown in the second scene is the set that remains after the action shown in the first scene is completed. Encourage them to think of 9 − 1 and 8 as names for the same number. Then they should decide that, since 9 − 1 and 8 are names for the same number, they can say "9 − 1 = 8." They should think of 8 as the standard name for 9 − 1. Then have a child read the sentence given in picture C.

Use a similar procedure for picture D on this page. Then discuss pictures E and F.

It is best to avoid use of such expressions as "8 balls plus 1 ball is equal to 9 balls." Remember that "plus," "minus," and "is equal to" are properly used only with numbers.

STA 2 page 70
Workbook edition page 100

Basic facts: sums and minuends of 9

The children continue to learn the standard names for phrases that express action situations in which the original set or the combined set is a set of 9 objects.

For each picture, have the children describe what is happening in the first scene. Next, ask them to give the appropriate phrase (9 − 4 for picture A; 7 + 2 for picture C; and so on). Be sure the children understand that the second scene shows either the combined set or the set remaining after the action shown in the first scene has been completed. Then ask for the standard name for the number. The children should discover the standard name by examining the second scene and determining the numerousness of the pictured set. This should be done by grouping the objects, not by counting. Allow the children to use markers to illustrate the action shown in the picture. Finally, have the sentence in the picture read aloud.

The teaching suggestions given for using *STA 2* page 69 (Wkbk page 99) may be adapted to this page.

STA 2 page 71
Workbook edition page 101

Basic facts: sums and minuends of 9

The children continue to learn the standard names for phrases that express action situations in which the original set or the resulting set is a set of 9 objects.

For each picture, have the children describe what is happening in the first scene. Next, ask them to give the appropriate phrase (6 + 3 for picture A; 9 − 3 for picture C; and so on). Then have them give the standard name for the number. They should discover the standard name by examining the second scene and determining the numerousness of the pictured set. Allow the children to use markers and to illustrate

the action shown in the picture. Finally, have the sentence in the picture read aloud.

For pictures E and F, the children should first determine the numerals that replace the square screen and the round screen and then give the completed sentence.

The teaching suggestions given for using *STA 2* page 69 (Wkbk page 99) may be adapted to this page.

STA 2 page 72

Basic facts: sums and minuends of 9

The children have studied all the basic facts with sums and minuends of 9. Now they apply what they have studied.

Notice that on this page a single scene, rather than two-scene picture, is used. For each picture that shows additive action, the children should observe the joining action and should determine the numerousness of the resulting set. For each picture that shows subtractive action, they should observe the separating action, and they should determine the number of objects in the original set.

Allow the children to use markers to illustrate the action shown in a picture. Each child should have a set of cards, each card showing one of the following symbols: +, −, =, 1, 2, 3, 4, 5, 6, 7, 8, 9. The children could use these cards to build sentences that express the various situations.

For each picture, have the children first describe what is happening. For picture A, they should observe that 4 drums are being pushed toward 5 drums. Next, they should decide which word, "plus" or "minus," can be used to express the action. Then they should give the sentence that expresses the situation. For picture A, they should select the word "plus" and give the sentence "Five plus four is equal to nine."

After each picture has been discussed in this way, direct attention to the sentences printed in the pictures. Have the children decide what numeral replaces each placeholder. Then have them take turns reading the sentences aloud, making the appropriate replacements.

STA 2 page 73

Basic facts: sums and minuends through 9

The children apply what they have learned about the addition and subtraction basic facts with sums and minuends through 9.

Direct attention to picture A. The children should observe that 6 ducks are moving away from 3 ducks. Ask how many ducks were in the original set. [9] Next ask what word can be used to express the action. [minus] Then have a child give the sentence "Nine minus six is equal to three."

After discussing picture A, you may wish to have the children use markers to show the action. You also could have them use their cards to build the appropriate sentence.

Follow a similar procedure for each of the other pictures.

After all the pictures have been discussed, direct attention to the exercises at the right. For each exercise, have the children read the exercise silently and decide what numeral replaces the placeholder. Then have one child read the sentence aloud, supplying the correct numeral.

Each of the pictures on page 73 shows an action that is expressed by one or more of the sentences that appear at the right of the page. As another activity, have the children find the pictures that relate to particular sentences. (For example, picture C and sentence J)

If further practice seems necessary, select pictures at random. For each picture, have the children find the sentence (or sentences) at the right that expresses the pictured situation. As an example, for picture A, the children could give sentences G and Z. In using this activity, be sure the children understand that the sentence 4 + 5 = 9 relates to a physical

situation that is different from the situation related to the sentence 5 + 4 = 9.

When you have completed the work with this page, you could give the children oral instructions such as "Subtract 2 from 9" and "Add 3 to 6" and have them use their cards to build appropriate sentences. The children should first build a sentence containing a placeholder (9 − 2 = ▩) and then should replace the placeholder with the correct numeral (9 − 2 = 7). Use this activity to reinforce their learning of the basic facts that have already been taught.

Practice tablet page 52
Workbook edition page 102

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 1 + 8 = 9 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

Either before the children begin to work, or after they have completed Pictures A through H, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

Practice tablet page 53
Workbook edition page 103

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 9 − 8 = 1 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

Either before the children begin to work, or after they have completed Pictures A through H, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

Practice tablet page 54
Workbook edition page 104

Say: "Read each problem carefully and think about what is happening in the problem. Next, make the sentence that describes the problem and write this sentence on the first response space. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is. Be sure you read each problem carefully because some problems are about joining action and other problems are about separating action."

Either before the children begin to work, or after they have completed Problems A through F, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

STA 2 page 74
Workbook edition page 105

Basic facts: sums and minuends of 10

The children learn the standard names for phrases that express situations in which the resulting set or the original set is a set of 10 objects.

For each picture have the children describe what is happening in the first scene. Next, ask them to name the phrase that expresses the situation. Be sure the children understand that the second scene pictures either the combined sets or the remaining set after the action shown in the first scene has been completed. Then ask for the standard name for the number. Encourage the children to discover the standard name by examining the second scene and determining the numerousness of the pic-

tured set by grouping the objects, not by counting. If necessary, allow the children to use markers. Finally, have the sentence in the picture read aloud.

The teaching suggestions given for using *STA 2* page 69 (Wkbk page 99) may be adapted to this page. Provide each child with a card for the numeral "10."

STA 2 page 75
Workbook edition page 106

Basic facts: sums and minuends of 10

The children continue to learn standard names for phrases that express situations in which the resulting set or the original set is a set of 10 objects.

For each picture, have the children describe what is happening in the first scene. Next, ask them to give the phrase that expresses the situation (8 + 2 for picture B; 10 − 2 for picture D). Then ask them to give the standard name for the number. The children should discover the standard name by examining the second scene and determining the numerousness of the pictured set. Allow the children to use markers to illustrate the action shown in the picture. Finally, have the sentence in the picture read aloud.

The teaching suggestions given for using *STA 2* page 69 (Wkbk page 99) may be adapted to this page.

STA 2 page 76
Workbook edition page 107

Basic facts: sums and minuends of 10

The children continue to learn standard names for phrases that express situations in which the original set or the resulting set is a set of 10 objects.

The lesson notes for *STA 2* pages 74 and 75 (Wkbk pages 105 and 106) and the teaching suggestions given for using *STA 2* page 69 (Wkbk page 99) may be adapted to this page.

STA 2 page 77

Basic facts: sums and minuends of 10

The children have studied all the basic facts with sums and minuends of 10. Now they apply what they have studied.

For each picture that shows additive action, the children should observe the joining action. For each picture that shows subtractive action, they should observe the separating action. Have them describe what is happening in each picture and give a sentence that expresses the situation.

After each pictorial situation has been discussed, direct attention to the sentence printed in each picture. Have the children decide what numeral replaces each placeholder. Then have them take turns in reading the sentences aloud, making appropriate replacements.

The teaching suggestions given for using *STA 2* page 72 may be adapted to this page.

STA 2 page 78

Basic facts: sums and minuends through 10

The children reinforce their learning of the basic facts with sums and minuends of 2 through 10.

For each picture, have the children describe what is happening and give a sentence that expresses the situation. For picture A, as an example, they should say "Ten minus one is equal to nine."

After all the pictures have been discussed, direct attention to the exercises at the left. For each exercise, have the children read the sentence silently and decide what numeral replaces the screen. Then have one child read the sentence aloud, supplying the correct numeral.

The teaching suggestions given for using *STA 2* page 78 may be adapted to this page.

Practice tablet page 55
Workbook edition page 108

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 8 + 2 = 10 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

Either before the children begin to work, or after they have completed Pictures A through H, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

Practice tablet page 56
Workbook edition page 109

Call attention to Picture A. Say: "Give the arithmetic sentence that tells what is happening in this picture. Write 3 + 7 = 10 on the response space. For each of the other pictures, you are to write the arithmetic sentence that tells what is happening in the picture."

Either before the children begin to work, or after they have completed Pictures A through H, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

STA 2 page 79

Keeping skillful: situations involving additive and subtractive action -- sentences of the forms 3 + 2 = ▮ and 6 - 4 = ▮ ; sums and minuends through 10

The children analyze and solve verbal problems that involve either additive action or subtractive action. They reinforce their learning of addition and subtraction basic facts that they have studied so far.

You will find that some of the children can complete independently the work on this page. For each problem, have these children write two sentences on their papers opposite the labeling letter. The first sentence should express the problem situation and should contain a placeholder. The second sentence should contain the numeral that replaces the placeholder. For problem A, they should write the following:

A 6 - 5 = ▮ 6 - 5 = 1

The children who cannot work independently should participate in activities similar to those you have used in teaching previous problem-solving pages. First, each problem should be read silently. Next, the children should show the action with markers and use their cards to build the mathematical sentence that shows what is happening. After all the problems have been worked through in this way, have the children reread each problem and find in the first block of exercises (blue letters A through L) the sentence that describes what is happening in the problem. For problem A they should select sentence E (6 - 5 = ▮) as the appropriate sentence. Ask a child to reread the sentence, supplying the correct replacement, and then to give the answer to the problem.

Both blocks of exercises at the right (blue letters A through L; brown letters A through M) may be used for oral or written practice. First have the children take turns in reading the exercises. Then they might respond orally, in turn, giving the sentence again with the correct numeral for the placeholder, or they might write the completed sentence on a sheet of paper. If the responses are written, supply the correct responses so that the children can verify their work when they have completed the page.

STA 2 page 80

Keeping skillful: situations involving additive and subtractive action -- sentences of the forms 3 + 2 = ■ and 6 - 4 = ■ ; sums and minuends through 10

The children analyze and solve verbal problems that involve either additive action or subtractive action. They reinforce their learning of addition and subtraction basic facts that they have studied so far.

The teaching suggestions given for using *STA 2* page 79 may be adapted to this page.

Practice tablet page 57
Workbook edition page 110

Say: "Read each problem carefully and think about what is happening in the problem. Next, make the sentence that describes the problem and write this sentence on the first response space. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is. Be sure you read each problem carefully because some problems are about joining action and other problems are about separating action."

Either before the children begin to work, or after they have completed Problems A through F, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

Practice tablet page 58
Workbook edition page 111

Call attention to Picture A. Say: "For each picture, you are to name each segment that is shown in the picture. Then you write the two letters that name the segment on one of the response spaces."

Practice tablet page 59
Workbook edition page 112

Notice that this page consists of two blocks of exercises. Use your own judgment as to how many of these exercises you can assign at one time.

Tell the children that they are to complete each of the exercises by writing the correct numeral on the response space.

NUMERATION SYSTEM

A numeration system is concerned with *numerals*, the names for numbers. A number is an idea that is associated with a set. Because a number is a mathematical concept, it cannot be seen, written, or spoken. To express numbers symbolically, it is necessary to have an organized set of symbols. The numeration system in common use today employs the ten basic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These symbols and two basic principles, *grouping* and *place value*, enable us to write a number name for every set of objects.

The principle of grouping is a fundamental idea that is used with most effective numeration systems in current use. The numeration system most commonly used at the present time employs the principle of grouping by tens and multiples of ten. This system is called a base-ten numeration system, or a decimal numeration system. The principle of place value is another fundamental idea used in effective numeration systems and in the base-ten numeration system. Place value makes the meaning of each digit in a numeral dependent upon its position within the numeral. These two ideas, grouping and place value, provide the foundation for the base-ten numeration system.

For a child to understand names for numbers greater than 10, it is essential that he understand the basic principles of the base-ten system. To understand a given numeral, he must associate this numeral with one and only

one number. To understand its written notation (the numeral 135, for example), he must understand grouping by hundreds (10 tens), tens, and ones (135 is 1 hundred 3 tens 5 ones). He must also understand the idea of place value. In the numeral 135, 5 is in the ones' place and means five ones; 3 is in the tens' place and means three groups of ten; 1 is in the hundreds' place and means one group of one hundred.

In grade 1, the children studied the natural numbers less than 100. In grade 2, they extend their knowledge of the natural numbers to include the numbers less than 100. The natural numbers are the numbers expressed by the numerals 0, 1, 2, and so on.

Notice the different ways that are used to show groups of 10 and groups of 100 (10 groups of 10) throughout this section. Observe the use of "pyramids," bags, bundles, boxes, bunches, and jars. Work toward having the children accept an arrangement as a group of 10 (or 100, as the case may be) without counting the individual objects.

STA 2 page 81
Workbook edition page 113

Grouping; use of tally marks and numerals to represent the groupings; place value [reteaching]

The children have learned to recognize the numbers associated with sets that have from 1 through 10 members. Now they work with sets that have many members. They learn to record the base-ten numeral associated with each set.

When the children open their books to this page, let them identify the various objects shown in the nine pictures — marbles, lollipops, logs, balls, pencils, pieces of candy, paint brushes, boxes, and candy mints. Lead the children to observe that the objects have been grouped by tens and that each picture shows groups of ten objects and single objects.

Call attention to the various devices used to show a group of ten. Work toward having the children accept, without counting, groups of 10 that are in bundles, bags, boxes, or arranged in the shape of a pyramid. For example, the bags of marbles should be accepted as containing 10 marbles each, the boxes of lollipops as containing 10 lollipops each, and so on.

Call attention to picture A. Use questions to help the children determine that there are 2 groups of ten marbles each and 6 single marbles. Have them study the "box" shown in the picture. Explain that we call this "box" a *tally box*. Then call attention to the marks in the tally box and tell the children that we call each of these marks a *tally mark*. Say: "Notice that the tally box has two parts. One part is a place for ones, and we call this part the *ones' place*. Why do we have another part? What name can we give to this other part?" [The other part is a place for tens. We call it the *tens' place*.]

Ask: "How many single marbles are there? We should have a tally mark for each single marble. Where are these tally marks? Is there a tally mark in the ones' place of the tally box for each single marble? Why are there two tally marks in the tens' place of the tally box?" [There is a tally mark for each group of ten marbles.]

Say: "Look at the numerals that are written in the tally box. What numeral is in the tens' place? What numeral is in the ones' place? What does each of these numerals mean?" [The "2" in the tens' place tells us that there are two groups of ten marbles. The "6" in the ones' place tells us that there are six single marbles.] Lead the children to understand that the numerals "2" and "6" and the tally marks in each part of the tally box serve the same purpose.

At this time the children should read the numeral 26 as "2 tens 6 ones." Reading a numeral in this way helps them understand that a

two-digit numeral, such as 26, represents a number of tens and a number of ones. If a child reads "26" as "twenty-six," ask him to tell what twenty-six means and to show this with markers. He should say that twenty-six means 2 tens 6 ones.

Proceed in this same way with pictures B and C. With some children it may be necessary to use objects arranged in piles or bundles of 10, with extra single objects, to reinforce the learning based on the pictures.

For each of pictures D, E, and F, the children should decide that the numeral in the place for the tens in the tally box tells how many groups of 10 objects there are, and the numeral in the place for the ones tells how many single objects there are. Then have the children tell how many objects are pictured. For picture D, they should say "5 tens 7 ones."

For pictures G, H, and I, have the children use their cards labeled with numerals to indicate how many groups of 10 objects are pictured and how many single objects are pictured. Then have them tell how many objects are pictured. For picture G, they should say "7 tens 4 ones."

For the children who have difficulty with pictures D through I, draw a large tally box on the chalkboard, or have the children draw tally boxes on a sheet of paper. Then, for each picture, have them place tally marks in the appropriate places.

Provide sets of blocks, spools, and other small objects and give the children many opportunities to group such objects by tens. Then have them show, by writing tally marks and numerals in tally boxes, the number of objects in each set.

Practice tablet page 60
Workbook edition page 114

Direct attention to Picture A. Say: "How many tally marks should you put in the tens' space? [4] Put 4 tally marks in the tens' space. How many tally marks should you put in the ones' space? [None] What numeral should you write in the tens' space? [4] Write the numeral 4 in the tens' space. What numeral should you write in the ones' space? [0] Write the numeral 0 in the ones' space.

"For each of the other pictures, put a tally mark in the tens' space for each group of ten and a tally mark in the ones' space for each single object. Then, in the tens' space, write the numeral that tells how many tens there are. In the ones' space, write the numeral that tells how many ones there are."

Practice tablet page 61
Workbook edition page 115

Say: "For each picture, put a tally mark in the tens' space for each group of ten and a tally mark in the ones' space for each single object. Then, in the tens' space, write the numeral that tells how many tens there are. In the ones' space, write the numeral that tells how many ones there are."

Practice tablet page 62
Workbook edition page 116

Say: "For each picture, put a tally mark in the tens' space for each group of ten and a tally mark in the ones' space for each single object. Then, in the tens' space, write the numeral that tells how many tens there are. In the ones' space, write the numeral that tells how many ones there are."

STA 2 page 82
Workbook edition page 117

The decade numbers; oral use of decade names [reteaching]

The children learn the meaning and order of the decade numbers (10, 20, 30, etc.). They also learn the names and symbols for these

numbers. Have the children identify and talk about the various objects shown in the pictures --marbles, jacks, balls, pencils, etc. Have them notice the devices used to group the objects. Continue to work toward having the children accept, without counting, each of these groups as a group of ten.

Note that each picture shows only groups of 10. No single objects are pictured. Picture A shows 1 ten, B shows 2 tens, and so on.

Now direct attention to picture A. Help the children understand that the tally mark and the numeral in the tens' place each represent the one box of 10 balls. Ask: "Why are there no tally marks shown in the ones' space? [There are no single objects pictured.] What word do we use for the numeral in the ones' place of the tally box? [Zero] What does the numeral 0 tell us? [That there are no ones]"

Then direct attention to the numeral below the tally box. Explain that the "1" in the numeral 10 shows that there is 1 group of 10 objects and that the "0" shows that there are no single objects. After the children read the numeral 10 as "1 ten 0 ones," ask if anyone knows another way to read this numeral. Be sure they understand that "ten" has the same meaning as "1 ten 0 ones."

Follow a similar procedure for each of the other pictures. Introduce each of the other decade names in a similar manner.

After discussing all of the pictures, have the children point to each numeral, in order, and repeat each decade name. Then call the names in random order, and have the children point to the appropriate numeral.

As an added activity, provide each child with a set of nine cards on which the numerals 10, 20, 30, etc., have been written, and have the children arrange these cards to show the numerals in order. Then have them read the names of the numerals, "ten, twenty, thirty," and so on. Work toward having the children give the decade names in order. Use decade names, such as "thirty," only orally throughout the discussion of this page.

Practice tablet page 68
Workbook edition page 119

Direct attention to Picture A. Say: "What numeral should you write in the tens' space? [5] Write 5 in the tens' space. What numeral should you write in the ones' space? [0] Write 0 in the ones' space. What is the standard name for 5 tens 0 ones? [50] Write 50 on the response space.

"For each of the other pictures, write the numeral for the number of tens in the tens' space. In the ones' space, write the numeral for the number of ones. On the response space, write the standard name for the number."

Either before the children begin to work or after they have completed Pictures A through I, direct attention to the response spaces at the right. Say: "You are going to write the numerals 10, 20, 30, and so on, in order. Trace the first two numerals. Then write the other numerals in order. Write one numeral on each response space."

STA 2 page 83
Workbook edition page 119

Order of ones within a decade [reteaching]

The children discover that, in the second decade ("the teens"), the digit (any of the numerals 0 through 9) in the tens' place remains the same, while the digit in the ones' place changes successively from 0 through 9. They learn that this pattern exists within each decade.

For this page have the children identify the various objects and discuss the groupings shown in the pictures. They should discover that, in each picture, there is shown 1 group of 10 objects and a number of single objects. Help them discover that pictures B, C, D, and

so on, each show one more single object than is shown in the preceding picture.

Direct attention to picture A. The children should understand that, in the numeral "10," the "1" represents 1 group of 10 and the "0" indicates there are no ones. Have them name the numeral 10 as "ten."

For picture B, have the children tell how many groups of 10 and how many single objects are pictured. Help them to understand that the tally mark and the numeral "1" in the tens' place each represent a single group of 10 and that the single tally mark and the numeral "1" in the ones' space each represent a single object. Next ask the children to look at the numeral below the tally box. Explain that "eleven" is another name for 1 ten 1 one. Emphasize the fact that the number 11 is one greater than the number 10.

Use a similar procedure for each of pictures C through J. Give special attention to the names for the numbers 11 and 12. These may cause verbal difficulty for the children because they do not have the same verbal form as thirteen, fourteen, fifteen, and so on.

Be sure the children know what each digit within a numeral represents. For example, in 16, the "1" means 1 group of 10, the "6" means 6 single objects.

After discussing all of the pictures, have the children point to each numeral, in order, and give its word name. Then call the names of these numbers in random order and have the children point to the appropriate numerals in their books. They could also use their cards labeled with numerals to show the numbers named. Finally, they should name the numbers in order and tell what each numeral means.

Have available various objects or cutouts for a flannel board or draw on the chalkboard pictures of 19 or fewer objects. Have the children count the objects. Let them take turns counting from 1 to 19, 1 to 15, etc. Ten of the objects could be grouped together and the counting started from 10.

STA 2 page 84
Workbook edition page 120

Order of ones within a decade [reteaching]

The children have learned that in the second decade ("the teens") the digit in the tens' place remains the same and the digit in the ones' place repeats the sequence from 0 through 9. Now they apply this idea to the numbers in the third decade.

The children should begin to understand that the order of ones within a decade is the same for any given decade. Any decade could have been selected to reinforce this idea. The thirties were selected because the children can recognize 3 groups of 10 at a glance.

Lead the children to discover that each picture on the page shows 3 groups, each of which has 10 objects. Ask what name is given to 3 groups of 10. Then, for each picture, have the children tell how many groups of 10 and how many single objects are shown. Next, ask them to give the name of the number associated with the particular set of objects. For picture A, the children should first say that there are 3 tens 7 ones; then they should say that there are thirty-seven logs.

After all of the pictures have been discussed, ask the children to find the picture of the set that has the least number of objects (picture J). Have them name the number associated with this set (thirty). Then direct attention to the numerals given at the left of the page. Have the children identify the numeral that names the number thirty.

Next have them find the picture of the set that has one more object than the set of thirty objects, name the number associated with this set (thirty-one), and identify the numeral that names this number. Continue in this way until all pictures have been discussed and the numbers ordered.

Give the children opportunities to order the ones within other decades through the nineties. They should understand that the order of ones is the same for any decade.

Practice tablet page 64
Workbook edition page 121

Call attention to Picture A. Say: "How many tens are there? [5] How many ones are there? [5] What is the standard name for 5 tens 5 ones? [55] Write 55 on the response space.

"For each of the other pictures, decide how many objects are shown in the picture. Then, on the response space, write the numeral that tells how many objects there are."

Either before the children begin to work or after they have completed Pictures A through J, direct attention to the response spaces at the right. Say: "You are going to write the numeral 50, 51, 52, and so on, in order. Trace the first two numerals. Then write the other numerals in order. Write one numeral on each response space."

Practice tablet page 65
Workbook edition page 122

Direct attention to Picture A. Say: "First you are to find the standard name for each of the four sets of objects. Notice the response space below each set. On each response space, write the standard name of the set shown. Look at the four response spaces at the end of the picture. You are to write the four standard names in order. Begin with the numeral that names the least number of objects.

"Do each of the other pictures in the same way."

Practice tablet page 66
Workbook edition page 123

Call attention to Picture A. Say: "How many tens are there? [7] How many ones are there? [8] What is the standard name for 7 tens 8 ones? [78] Write 78 on the response space.

"For each of the other pictures, decide how many objects there are. On the response space, write the numeral that tells how many objects there are."

Practice tablet page 67
Workbook edition page 124

Say: "For each picture, decide how many objects there are. On the response space, write the numeral that tells how many objects there are."

STA 2 page 85
Workbook edition page 125

Naming the number associated with a set [reteaching]

The children are familiar with the idea that a set of objects can be organized as groups of 10 and single objects. They also know how to name the number associated with a set of objects. Now they apply what they have studied about the principles of grouping and place value.

For picture A, have the children tell how many tens and how many ones are shown. Then direct attention to the expression that is printed below the objects. Have the children read the expression and help them discover that this expression states how many tens and how many ones are shown in the picture. Then have them name the number associated with the objects. Relate the numeral "28" to the pictured objects and the expression "2 tens 8 ones." They should read the numeral 28 as "twenty-eight." Be sure they understand that "2 tens 8 ones" and "twenty-eight" name the same number.

For picture B, the children should first decide how many tens and how many ones are pictured and should read the expression given. Then they should name the number. Have them show the numeral that replaces the screen by using numeral cards or writing the numeral.

For each of pictures C through F have the children decide how many objects are shown,

what numeral replaces each placeholder, and then use numeral cards to show the correct numerals; or they may write the numerals.

Now direct attention to the exercises at the side of the page. The responses for these exercises may be given orally or may be written. For the first block of exercises, the children should use numeral cards, or write the numeral, to show the correct replacement for the square screen. For the second block of exercises, they should use cards, or write the numerals, to show the correct replacement for each screen.

Practice tablet page 68
Workbook edition page 126

Notice that this page consists of two blocks of exercises. Use your own judgment as to how many exercises to assign at one time.

Tell the children that they are to complete each of the exercises by writing the correct numeral on the response space.

STA 2 page 80
Workbook edition page 127
Grouping by hundreds; introduction of the printed word "hundred" and the symbol "100"

The children have learned to organize sets of objects into groups of 10 and to identify and write the numerals for the numbers associated with these sets of objects. Now they learn to think of a set of 10 groups of ten objects as a single group of one hundred objects. They also learn to recognize the printed word "hundred" and the symbol "100."

In the base-ten numeration system, any number, no matter how great, can be expressed by a numeral. Furthermore, any numeral in the base-ten system of numeration may be written by using only the 10 symbols "0" through "9" and the principles of the numeration system. To name the number associated with a set of 10 tens in the counting process, an entirely new word is used, namely, *hundred*. By using this new word, along with those already in use, it is relatively easy to read numerals through 999. Thus the numeral 437, which could be read as "forty-three tens seven ones," is usually read "four hundred thirty-seven." The reading or the oral description of numbers greater than 99 would be lengthy and awkward if the word "hundred" were not introduced.

Children must learn to use the word "hundred" in order to talk intelligibly about numbers greater than 99. They should know the word "hundred" not only as the name of the number that is one greater than ninety-nine, but also as the name of a very special group -- the group of 10 tens. They should learn the significance of place value for hundreds in written numerals. In short, their knowledge of the numeration system must be extended to cover the range from 99 to 999. The objective is not to have the children learn 900 numerals as more or less isolated numerals, but to learn the *basic principles of the numeration system.*

Direct attention to picture A. Have the children describe what they see. Be sure they understand that each bundle contains 10 sticks. Next have them count the number of sticks by tens, counting "10, 20, 30, 40," and so on. When they have said "90," ask if anyone knows what words are used for 10 bundles of ten each. If no one knows the word, introduce the words "one hundred." Explain that 10 groups of ten can be thought of as a single group and that the name of this group is "one hundred." Introduce the printed word "hundred" and the symbol "100" as names for a set of objects having 10 groups of ten objects. Ask the children to again count the bundles of sticks. This time have them read the numeral printed under each bundle.

For pictures B through G, have the children talk about the various objects that are shown

and how these objects are grouped. Encourage them to accept the devices showing a group of 100 objects and the devices showing a group of 10 objects without counting.

For each picture, have the children first decide how many hundreds or tens are pictured. Then ask them to read and answer the question given in each picture.

STA 2 page 87
Workbook edition page 128
Grouping by hundreds, tens, and ones; use of tally marks to represent the groupings; place value (three places)

The children have learned to recognize the number associated with sets having 10 or fewer objects. For sets that have many members, they have learned to group the objects by tens. Now they learn to think of a set that has 10 groups of ten objects in each group as a set of 100 and to use tally marks to record the name of the number associated with the set.

Notice that there are two teaching sequences on this page. Pictures A, B, and C make up one teaching sequence. Pictures D, E, and F make up the other teaching sequence.

For picture A, the children should observe that boxes each containing ten cans and a number of single cans are shown. They should discover, by counting, that there are 14 boxes of 10 cans each and that there are as many tally marks in the tens' place of the tally box as there are boxes. Stress the fact that there is a tally mark for each set of ten cans. They should observe that 3 single cans are shown and should understand that there is a tally mark for each of the single cans in the ones' place.

Lead the children to understand that it is difficult to determine the number of cans because the number of boxes is so great. Also, emphasize the fact that it is difficult to determine the number of tally marks in the tens' place of the tally box. Ask if anyone can suggest a way to simplify recognition of the number of boxes and the number of tally marks. If no one suggests a way, direct attention to picture B. Explain that we can group the cans and the tally marks into groups of 10 tens. Call attention to the rings that have been drawn around 10 of the boxes and around 10 of the tally marks in the tens' place. Remind the children that there are some single boxes and some tally marks in the tens' place of the tally box that are left over.

For picture C, the children should observe that the group of ten boxes have been grouped in a definite arrangement. Be sure the children understand that there are 10 boxes, each containing ten objects, in the "pyramid." Ask the children what they have learned to call a group of 10 tens. They should recall that a group of 10 tens is called "hundred." Have the children look at the tally box. Ask: "How many parts are there in this tally box? What does each tally mark in the new part mean? What name can we use for the new part?" [There are three parts. Each tally mark in the new part represents a group of one hundred objects. We can call this part the hundreds' place.] Ask the children how many groups of ten boxes there are and how many tally marks are in the hundreds' place. Then ask how many single boxes, each having 10 cans, there are and how many tally marks are in the tens' place. Finally, ask how many single cans there are and how many tally marks are in the ones' place. It is essential that the children understand that each tally mark in the hundreds' place represents a set of ten groups of 10 objects; that each tally mark in the tens' place represents a set of ten objects; and that each tally mark in the ones' place represents a set of 1 object. To tell how many objects there are, the children should, at this time, say "1 hundred 4 tens 3 ones," not "one hundred forty-three."

Avoid asking the children to read numerals for numbers between 100 and 1000 until the

names for these numbers have been introduced. Encourage children to read a numeral such as 268 as "2 hundreds 6 tens 8 ones." Reading a numeral in this way reinforces their understanding of the idea that a three-digit numeral represents a number of hundreds, a number of tens, and a number of ones.

Use a similar teaching procedure for pictures D, E, and F.

STA 2 page 88
Workbook edition page 129
Grouping; use of tally marks; place value (three places)

The children continue to work with sets of objects grouped as hundreds, tens, and ones, and to use tally marks to express a numeral for the number associated with the set.

For each of pictures A, B, C, and D, ask the children to tell how many groups of 100 are pictured. Then ask them to determine if there are as many tally marks in the hundreds' place of the tally box as there are groups of 100. Next, ask them to tell how many groups of 10 are pictured and to determine if there are as many tally marks in the tens' place as there are groups of 10. Finally, have them tell how many single objects are pictured and determine if there are as many tally marks in the ones' place as there are single objects.

Call attention to the words "hundreds," "tens" and "ones" printed above the tally box. Emphasize that each tally mark for a group of 100 must be in the hundreds' place of the tally box; each tally mark for a group of 10 must be in the tens' place; and each tally mark for a single object must be in the ones' place. Then have the children tell the number of objects pictured. For picture A, they should say "4 hundreds 1 ten 4 ones."

For each picture in the bottom row, have the children give the following: how many groups of 100 are pictured and how many tally marks should be placed in the hundreds' place of the tally box; how many groups of 10 are pictured and how many tally marks should be placed in the tens' place; and how many single objects are pictured and how many tally marks should be placed in the ones' place. Finally, they should give the number (how many hundreds, tens, and ones) of objects pictured. You may wish to draw a large tally box on the chalkboard and have the children draw tally marks as each picture is discussed.

Practice tablet page 69
Workbook edition page 130

Direct attention to Picture A. Say: "How many groups of 100 sticks are there? [3] In the hundreds' space, put a tally mark for each group of 100 sticks. How many groups of 10 sticks are there? [6] In the tens' space, put a tally mark for each group of 10 sticks. How many single sticks are there? [1] In the ones' space, put a tally mark for each single stick.

"Do each of the other pictures in the same way. In the hundreds' space, put a tally mark for each group of 100. In the tens' space, put a tally mark for each group of 10. In the ones' space, put a tally mark for each single object."

STA 2 page 89
Workbook edition page 131

Grouping; use of numerals; place value (three places)

The children continue to study grouping by hundreds, tens, and ones. They learn to use numerals rather than tally marks to express the number associated with a set of objects.

Picture A shows the following: 3 groups, each group containing 10 boxes and representing 100 objects; a tally mark in the hundreds' place of the tally box for each group of 100

objects; 5 single boxes, each containing 10 objects; a tally mark in the tens' place for each single box of 10 objects; 2 single objects; and a tally mark in the ones' place for each single object. They should observe that the tally marks in the tally box mean that there are 3 hundreds 5 tens 2 ones.

Explain that it is often easier to use a numeral than to use tally marks to show how many objects are pictured. Discuss the numerals in the tally box. Ask: "Why is the 3 in the hundreds' place? [3 groups of 100 objects each are shown.] Why is the 5 in the tens' place? [5 groups of 10 objects each are shown.] Why is the 2 in the ones' place? [2 single marbles are shown.] How many marbles are shown in picture A? [3 hundreds 5 tens 2 ones]" Follow a similar procedure for each of pictures B, C, and D.

For pictures E and F, have the children use cards labeled with numerals to indicate how many groups of 100, then how many groups of 10, and then how many single objects are pictured. Next have them tell how many objects are pictured. For picture E, they should say "4 hundreds 2 tens 8 ones."

For the children who have difficulty in responding to questions about pictures E and F, draw a large tally box on the chalkboard, or have the children draw tally boxes on a sheet of paper. Then, for each picture, have them place tally marks and numerals in the appropriate spaces.

Have available many sticks (many teachers use tongue depressors) that can be tied into bundles of 10 each. Between 100 and 200 of these objects and enough rubber bands to fasten them in bundles are needed. Good use can be made of boxes that are alike in size and shape and large enough so that each will hold 10 objects. Give the children many opportunities to group such objects and to show, by writing numerals in tally boxes, the number of objects in each set.

Practice tablet page 70
Workbook edition page 132

Direct attention to Picture A. Say: "What numeral should you write in the hundreds' space? [8] Write 8 in the hundreds' space. What numeral should you write in the tens' space? [3] Write 3 in the tens' space. What numeral should you write in the ones' space? [0] Write 0 in the ones' space.

"Do each of the other pictures in the same way. In the hundreds' space, write the numeral that tells how many groups of 100 there are. In the tens' space, write the numeral that tells how many groups of 10 there are. In the ones' space, write the numeral that tells how many single objects there are."

If some children have difficulty deciding what numeral belongs in each space, have them continue to use both tally marks and numerals.

Practice tablet page 71
Workbook edition page 133

Say: "Do each picture in the same way. In the hundreds' space, write the numeral that tells how many groups of 100 there are. In the tens' space, write the numeral that tells how many groups of 10 there are. In the ones' space, write the numeral that tells how many single objects there are."

Practice tablet page 72
Workbook edition page 134

Say: "Do each picture in the same way. In the hundreds' space, write the numeral that tells how many groups of 100 there are. In the tens' space, write the numeral that tells how many groups of 10 there are. In the ones' space, write the numeral that tells how many single objects there are."

STA 2 page 90
Workbook edition page 135

Order of the hundreds

The children learn about the order of the hundreds (100, 200, 300, etc.). They also learn the word names and the symbols for these numbers.

Have the children discuss each of the pictures on this page. Each picture shows only groups of 100. No groupings of tens and no single objects are shown. They should accept each group as 100, without counting. Ask them to tell how many groups of 100 are shown in each picture. They should say: "Picture A shows 1 hundred;" "Picture B shows 2 hundreds," and so on.

Direct attention again to picture A. Lead the children to understand that both the tally mark and the numeral in the hundreds' place of the tally box represent the single group of 100 objects pictured in A. They should also understand that there are no tally marks in the tens' place because no groupings of 10 objects are shown in the picture, and that there are no tally marks in the ones' place because no single objects are pictured. Ask: "What does the numeral 0 in the tens' place mean?" [The zero indicates that there are no tens.] "What does the numeral 0 in the ones' place mean?" [That there are no ones.

Then direct attention to the numeral below the tally box. Have the children explain that the "1" in the numeral 100 shows that there is 1 group of 100 objects, the "0" beside the "1" shows that there are no groups of 10 objects, and the next "0" shows that there are no single objects. Then they should read the numeral 100 as "1 hundred 0 tens 0 ones" and as "one hundred." Be sure they understand that both ways of reading the numeral mean the same.

Follow a similar procedure for each of the other pictures. For picture B, as an example, let the children decide that instead of saying "2 hundreds 0 tens 0 ones," they can say "two hundred." Discuss the name for each of the hundreds in a similar manner.

After discussing all of the pictures, have the children point to each numeral, in order, and repeat each name aloud. Later they can use their numeral cards to show, or they can write, the numerals used to name groups of 100 that you state at random (for example, have them show the numeral that names 4 hundreds, that is, 400). Finally, they should be able to say the names of the hundreds in order and understand the meaning of each name. Throughout the discussion of this page, use names such as "five hundred" orally only.

Practice tablet page 73
Workbook edition page 136

Direct attention to Picture A. Say: "What numeral should you write in the hundreds' space? [2] Write 2 in the hundreds' space. What numeral should you write in the tens' space? [0] Write 0 in the tens' space. What numeral should you write in the ones' space? [0] Write 0 in the ones' space. What is the standard name for 5 hundreds 0 tens 0 ones? [500] Write 500 on the response space.

"Do each of the other pictures in the same way. In the hundreds' space, write the numeral for the number of hundreds. In the tens' space, write the numeral for the number of tens. In the ones' space, write the numeral for the number of ones. Then on the response space, write the standard name for the number."

Either before the children begin to work or after they have completed Pictures A through I, direct attention to the response spaces at the right. Say: "You are going to write the numerals 100, 200, 300, and so on, in order. Trace the first two numerals. Then write the other numerals in order. Write one numeral on each response space."

STA 2 page 91
Workbook edition page 137

Order of decades within a hundred

The children learn that within a given hundred, the digit in the hundreds' place remains the same, while the digit in the tens' place changes successively from 0 through 9. The words "two hundred ten," "two hundred twenty," and so on are introduced orally.

It is important that the children begin to understand that the order of tens within a hundred is the same within any given hundred. The decades within any hundred could have been selected to present this idea. The second hundred was selected because the children can easily recognize two groups of 100 at a glance

Provide each child with a worksheet on which you have made 10 tally boxes. Each box should be labeled with a letter that corresponds to one of the pictures. Above each tally box should be written the words "hundreds," "tens," and "ones."

Have the children find the tally box labeled A on his worksheet, and direct attention to picture A. Ask: "How many groups of 100 are there? How many tally marks do we need to show this number of groups? In which place of the tally box do we put the 2 tally marks to show that there are 2 groups of 100? How many tens are shown in picture A? How many tally marks do we put in the tens' place? How many single objects are there? How many tally marks do we put in the ones' place? What numeral can we write in the hundreds' place to tell how many hundreds there are? What numeral can we write in the tens' place? What numeral do we write in the ones' place? How many cans are shown in picture A?" Write the numeral 210 on the chalkboard. Ask the children to explain what each of the digits means. Have them read the numeral 210 as "two hundred ten."

Use a similar procedure for each of the other pictures.

After all of the pictures have been discussed, ask the children to find the picture of the set with the least number of objects (picture G). Have them tell how many objects are in this set. Then call attention to the numerals given at the right of the page. Ask the children to identify the numeral that names the number of objects shown in G.

Next have them find the picture of the set that has one more group of ten objects than the set of two hundred objects, name the number of objects in this set (two hundred ten), and identify the numeral that names this number. Continue until all pictures have been discussed and the numbers ordered.

Be sure the children know what each digit (any of the numerals 0 through 9) within a given numeral represents. For example, in 270, the 2 means 2 groups of 100, the 7 means 7 groups of 10, and the 0 means no single objects.

Practice tablet page 74
Workbook edition page 138

Call attention to Picture A. Say: "How many crayons are there? [480] Write 480 on the response space.

"For each of the other pictures, decide how many objects there are. On the response space, write the numeral that tells how many objects there are."

Either before the children begin to work or after they have completed Pictures A through J, direct attention to the response spaces at the right. Say: "You are going to write the numerals 400, 410, 420, and so on, in order. Trace the first two numerals. Then write the other numerals in order. Write one numeral on each response space."

STA 2 page 92
Workbook edition page 139

Naming the number associated with a set

The children have learned the names of the hundreds and have ordered the hundreds and the decades within a hundred. Now they learn to name the number associated with a set that has from 100 through 999 members.

For each picture, the children first should tell how many groups of 100, how many groups of 10, and how many single objects are shown. Next they should name the number of objects. Then have them identify the numeral that names this number from the list of numerals at the left of the page.

After all the pictures have been discussed in this way, read aloud the name of a number associated with one of the pictured sets. Have various children identify the set by giving its letter. Continue giving names of numbers and having the children identify the pictured set that is associated with each number.

Ask the children to read the numerals listed in the column at the left. For example, they should read the numeral 871 as "eight hundred seventy-one." Help them to understand that the 8 represents 8 hundreds, the 7 represents 7 tens, and the 1 represents 1 one.

Practice tablet page 75
Workbook edition page 140

Say: "For each picture, decide how many objects there are. On the response space, write the numeral that tells how many objects there are."

Practice tablet page 76
Workbook edition page 141

Say: "For each picture, decide how many objects there are. On the response space, write the numeral that tells how many objects there are."

Practice tablet page 77
Workbook edition page 142

Say: "For each picture, decide how many objects there are. On the response space, write the numeral that tells how many objects there are."

STA 2 page 93

Naming the number associated with a set

The children are familiar with the idea that a set of objects can be organized as groups of 100, groups of 10, and single objects. They know the meaning of each digit within a three-digit numeral. On this page they apply what they have learned about place value and grouping in the base-ten numeration system.

For picture A, have the children decide how many hundreds, how many tens, and how many ones are shown. Then direct attention to the expression printed below the objects. The children should read the expression and discover that it tells how many objects are pictured. Then have them name the number of objects. They should read the numeral 258 as "two hundred fifty-eight." Ask questions to ensure that they understand that "2 hundreds 5 tens 8 ones" and "258" name the same number.

For picture B, the children should first decide how many hundreds, how many tens, and how many ones are pictured and should read the expression. Then they should name the number. Have them use cards labeled with numerals, or have them write numerals, to indicate the numerals that should replace the square screen.

For pictures C and D, have the children first decide how many objects are shown, name the numeral that replaces each placeholder, and then name the number associated with the set. As before, have them use numeral cards or write numerals to show the correct numerals.

After discussing all the pictures, direct attention to the exercises at the right of the page. For each exercise indicated by a red letter, the children should read the expression, name the number associated with the set, and use cards labeled with numerals, or write the numerals, to show the correct replacement for the square screen. For each exercise indicated by a blue letter, they should read the numeral, state how many hundreds, how many tens, and how many ones are in the set associated with the number, and use cards, or write numerals, to show the correct numeral.

For children who have difficulty with these exercises, have them use objects to build the set of objects indicated by a particular exercise.

Practice tablet page 78
Workbook edition page 143

Say: "In each exercise, you are given one name for a number. You are to find the standard name for this number. Complete each exercise by writing the correct numeral on the response space."

Practice tablet page 79
Workbook edition page 144

Say: "In each exercise, you are given the standard name for a number. You are to find another name for this number. Complete each exercise by writing the correct numeral on each response space."

Workbook edition page 145
Ideas of "greater than," "less than," and betweenness for numbers

The children extend their knowledge of the ideas of "greater than," "less than," and betweenness to include all numbers less than one thousand.

The blocks of exercises on this page may be used for oral or written practice. Notice that for exercises A through J (orange letters) there is more than one possible replacement for each placeholder.

If the responses are to be written, have the children first write the color of the letters in the block and then write the letters that are included in the block.

Then have the children read each exercise, decide what numeral (or numerals) replaces the placeholder, and write their response beside the appropriate letter. After the children have completed the exercises, supply the correct responses so that they can verify their work.

If the responses are to be given orally, have the children read the exercise silently and decide what numeral replaces the placeholder. Then have one child read the exercise aloud and give the correct replacement.

STA 2 pages 94 and 95
Workbook edition pages 146 and 147
Counting by fives, by tens, and by twenty-fives [extension]

The children have learned to count by ones and to group a set of objects. Now they learn about counting by fives, by tens, and by twenty-fives.

The counting chart includes numerals for the numbers from 1 through 200. For emphasis, numerals used in counting by fives are shown on yellow background (Wkbk, light blue); those used in counting by tens are enclosed by an orange box; and those used in counting by twenty-fives are enclosed by a blue box.

Direct attention to the chart. The children should observe that the chart includes the numerals from 1 through 200. Have them count by ones from 1 to 25, from 40 to 60, from 75 to 125, and so on. Explain that instead of counting by ones, we can count by fives, by tens, or by twenty-fives.

Have available the bundles of 10 objects that the children used during their discussion of the numeration system. Have them count sets of these bundles by tens. The number of objects counted can exceed 200. Then have them use the chart and count by tens from 10 to 90, from 80 to 120, from 100 to 200, and so on. Provide other opportunities for the children to count by tens.

Follow a similar procedure to introduce counting by fives and by twenty-fives. Sets of toothpicks, blocks, buttons, and other small objects, fastened together with rubber bands or placed in small boxes to indicate the groupings, are useful devices for counting purposes. Finally, direct attention to the exercises below the chart. For each of exercises A through P, the children should read an exercise silently and decide whether the exercise suggests counting by fives, by tens, or by twenty-fives. Then they should determine the numbers that complete the suggested sequence. Finally, have one child state the completed sequence aloud. For exercise A, he should say "5, 10, 15, 20, 25, 30, 35."

For each of exercises A through H, have the children read an exercise and count silently. Then have one child or a group of children give the suggested sequence aloud.

Let the children refer to the counting chart when necessary, but encourage them to learn to count by fives, by tens, and by twenty-fives without using the counting chart. Note that several of the exercises extend the range of numbers to be counted beyond the range of 200 shown on the chart. Provide other opportunities for the children to count by ones, by fives, by tens, and by twenty-fives.

Practice tablet page 80
Workbook edition page 143

Say: "On this page you are going to count by fives, by tens, or by twenty-fives. First decide by what number you are to count. Then on each response space, write the numeral you would say as you count."

Practice tablet page 81
Workbook edition page 149

Say: "On this page, you are given the directions for each exercise. Read these directions carefully. Be sure that you begin to count with the correct numeral. On each response space, write the numeral you would say as you count."

STA 2 page 96

See the teaching suggestions given for **Workbook edition** page 145

Practice tablet page 82
Workbook edition page 150

Notice that this page consists of two blocks of exercises. Use your own judgment as to how many exercises to assign at one time.

Tell the children that they are to complete each exercise by writing the correct numeral on the response space.

STA 2 page 97
Workbook edition page 151
Points, lines, segments, and polygons

The children apply the geometric ideas that they have studied, and they learn to recognize the printed words "point" and "polygon."

The exercises on this page may be used for oral or written practice. The children use the diagrams at the left to find the correct responses to each group of exercises.

Two new words, "point" and "polygon," are introduced on this page. If the responses are given orally, introduce each word as it occurs. If the responses are written, discuss and introduce the words "point" and "polygon" as

you would any new word before you assign the exercises.

If the responses are given orally, have the children read an exercise silently, study the diagram, and decide upon the response. Then ask various children to read the exercise aloud and to give their response. Be sure that the children understand the response to each exercise before proceeding to the next exercise.

If the responses are written, have the children write the letter of an exercise on their paper, read the exercise, study the diagram, and write their response beside the letter.

Practice tablet page 83
Workbook edition page 152

Say: "Read Exercise A. Use the picture to find your responses. Write your responses on the response spaces. Do each of the other exercises in the same way. For some exercises, you will not use all of the response spaces.

Practice tablet page 84
Workbook edition page 153

Say: "Read Exercise A. Use the picture to find your responses. Write your responses on the response spaces. Do each of the other exercises in the same way. For some exercises, you will not use all of the response spaces.

Practice tablet page 85
Workbook edition page 154

Say: "Read each problem. Think about what is happening in the problem. On the first response space, write the sentence that describes the problem. Use a screen to hold the place for the number you are to find. Then, on the second response space, write the sentence with the numeral that belongs where the screen is."

Either before the children begin to work, or after they have completed Problems A through F, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

MONEY

Among the practical uses of numbers are those associated with money. Children should learn to recognize the coins and know their names, such as 1 cent or "penny," 5 cents or "nickel," 10 cents or "dime," 25 cents or "quarter," 50 cents or "half dollar," and 100 cents or "dollar," and the relationships among them. They should also learn how to find values represented by sets of coins and to make change. The work in Grade 1 developed some of these understandings. The work in Grade 2 is directed toward reinforcing and extending these understandings.

Although the organization of our monetary system is related to the base-ten numeration system, our monetary system has some special features of its own. In particular, money involves grouping by fives as well as by tens. The nickel is equivalent in value to five pennies, the quarter to five nickels, and the fifty-cent coin to five dimes. These relationships are used in finding values represented by sets of coins and in making change. The idea of grouping by fives should be given special attention.

In some arithmetic programs, the knowledge that children are supposed to have about money is used as a basis for teaching the principles of the numeration system. However, the monetary system does not provide an appropriate basis for teaching the numeration system because it is really more complicated than the numeration system, since it involves grouping by both fives and tens. The principles of the numeration system should be introduced first and then

these principles applied to learning situations involving money.

The important problem of making correct change, or of checking to see if the correct change has been received, is similar to that of counting money. It involves additive thinking and is a process of counting upward from the amount owed to the amount paid. It is, of course, true that the correct amount of change can be calculated by the process of subtraction, but this is not the way change is commonly made in actual practice. Change making should be taught in the way it is most frequently met in actual situations.

In all activities connected with money, it is important that real coins be used whenever possible. Toy money is a poor substitute, since it lacks the appearance and the "feel" of the real thing. Familiarity with the coins depends to a great extent upon recognition of physical differences, and the pupil should have many opportunities to learn to distinguish among them.

STA 2 page 98
Workbook edition page 155
Recognition of money [extension]

The children learn to identify the penny, the nickel, the dime, the quarter, the half dollar, and the dollar.

The penny, nickel, dime, and quarter were introduced in *Seeing Through Arithmetic 1.* If you want to use illustrations of both sides of these coins, see page 88, *Seeing Through Arithmetic 1.*

Direct attention to the picture that shows the blue "coin" labeled with the numeral 1. Have several pennies available so that the children can compare real coins with the illustration of a real coin. Lead the children to discover that the coin is a cent, or penny. Introduce the printed words "cent" and "penny." Point out the symbolized coin. Explain that the numeral 1 tells them that this "coin" will be used in this book to represent a penny. Then discuss the nickel, dime, and quarter in a similar manner.

For the half dollar, have the children observe that both sides and the edge of the coin are shown. They should note the differences between the sides of the coin. Have several half dollars available so that the children can compare the real coins with the illustrations. Then tell the children that such a coin is a half dollar. Introduce the printed words "half dollar." Next discuss the symbolized coin.

Have the children examine the illustration of the dollar bill. Have several one dollar bills available so that the children can examine both sides of a one dollar bill. Have them note the differences between the two sides. Introduce the printed word "dollar" and call attention to the symbolized one dollar bill. Explain that the numeral 1 tells them that the "bill" is used to represent a one dollar bill.

Provide many opportunities for the children to examine, to handle, and to use real money as they study the following pages.

STA 2 page 99
Workbook edition page 156
Equivalent values [extension]

The children study the relationships among coins. They learn that a quarter has the same value as 25 pennies, as 5 nickels, and as 2 dimes 1 nickel. They learn that a half dollar has the same value as 50 pennies, as 10 nickels as 5 dimes, and as 2 quarters.

A nickel is not the same as 5 pennies, nor is a dime the same as 10 pennies. However, the value of a nickel is equivalent to the value of 5 pennies, and the value of a dime is equivalent to the value of 10 pennies. In class discussion, sentences similar to the

following could be used in talking about equivalent values:

a. _____ is just as much money as _____.

b. _____ will buy as much as _____.

c. _____ has the same value as _____.

Direct attention to picture A. Have the children identify the coins with the numeral 1 as pennies and the coin with the numeral 25 as a quarter. Point out the overlapping shown in the picture of 10 pennies. Encourage the children to accept a set of pennies shown in this manner as a set of 10, without counting. Then have them count the pennies as "10, 20, 21, 22, 23, 24, 25." Pose questions that will help them recall that each penny has the value of 1 cent. They should determine that each set of 10 pennies has a value of 10 cents and that the 25 pennies have a value of 25 cents.

Next, direct attention to the tally box. Have the children observe that the value of each set of 10 pennies is represented by a tally mark in the tens' place and that the value of each single penny is represented by a tally mark in the ones' place. Be sure they understand that the 25 pennies have a value of 25 cents. Then direct attention to the symbolized coin with the numeral 25 on it. They should notice that a value of 25 is indicated in the tally box by tally marks that show 2 tens 5 ones. They should discover that the tally marks represent the value of the quarter, as well as the value of the twenty-five pennies, and should decide that a quarter and 25 pennies have the same value. Point out that the 25 pennies can be replaced by the quarter. The replacement is shown by dimming the 25 pennies.

Follow a similar procedure for each of the the other pictures. The children should determine the value of the set of coins that is dimmed, identify the single coin, and then discover that the dimmed set of coins and the single coin have the same value. When discussing picture B, point out the overlapping and explain that 2 nickels shown in this manner should be accepted as a set with a value of 10. For picture B, the children should discover that 1 nickel has a value of 5 cents and that 5 nickels and 1 quarter have the same value.

For picture C, the children must understand that each dime has a value of 10 cents, and that 2 dimes and 1 nickel have the same value as 1 quarter; for picture D, 2 quarters and 1 half dollar have the same value; for picture E, 50 pennies and 1 half dollar have the same value; for picture F, 10 nickels and 1 half dollar have the same value; and for picture G, 5 dimes and 1 half dollar have the same value.

STA 2 page 100
Workbook edition page 157
Equivalent values [extension]

The children study the relationships between a dollar and various sets of coins. They learn that 1 dollar has the same value as 100 pennies, as 10 dimes, as 20 nickels, as 4 quarters, and as 2 half dollars.

Direct attention to picture A. The children should identify the coins as pennies and accept each grouping of pennies as a set of 10, without counting. They should count by tens and determine that 100 pennies are shown. Then direct attention to the tally box that shows two places. Have the children observe that the value of each set of 10 pennies is represented by a tally mark in the tens' place and that 10 tally marks are shown. They should see that a ring has been drawn around the 10 tally marks in the tens' place and should understand that this group can be organized as a single group of 100. Then direct attention to the tally box that shows three places. The children should understand that the single tally mark in the hundreds' place represents the same number of pennies as the 10 tally marks in the tens' place (100). Then they should decide that a set of 100 pennies has the value of 100 cents.

Now direct attention to the symbolized dollar bill. Have the children identify the bill that represents one dollar. They should discover that the tally mark in the hundreds' place represents the value of 1 dollar, as well as the value of 100 pennies, and should decide that 1 dollar and 100 pennies have the same value. Point out that the 100 pennies can be replaced by the 1 dollar. The replacement is shown by dimming the 100 pennies.

Follow a similar procedure for each of the other pictures. The children should determine the value of the set of coins that is dimmed and then discover that the value of the set of coins and the value of the 1 dollar bill are the same. For picture B, 10 dimes and 1 dollar have the same value; for picture C, 20 nickels and 1 dollar have the same value; for picture D, 4 quarters and 1 dollar have the same value; and for picture E, 2 half dollars and 1 dollar have the same value.

STA 2 page 101
Workbook edition page 158

Finding values represented by sets of coins [extension]

The children learn to find the value represented by a set of coins. They learn to recognize the symbol (¢) for "cent" or "cents" and the symbol for a dollar ($).

The terms "counting money" and "counting coins" are used often in everyday situations to refer to the process of finding the total value represented by a set of coins. The coins themselves are not counted; it is the value of each coin that is counted. Thus, the total value of the coins may be referred to as the total amount of the coins.

A child's understanding of the amount represented by a set of coins will be greater if, in counting the values of the coins, he uses his knowledge of the principles of the base-ten numeration system. The idea of organizing the coins so that their values may be counted by twenty-fives, tens, fives, and ones should be emphasized.

Before beginning work on this page, review counting by fives, by tens, and by twenty-fives. Then direct attention to picture A. Have the children identify each symbolized coin and state its value. Point out the two ways each value can be written and introduce the symbol (¢) for "cent" or "cents." Then the children should identify the symbolized dollar bill as representing 1 dollar. Direct attention to the four ways in which the value of a dollar may be written, and introduce the dollar sign ($).

Ask the children to look at picture B. Have them identify the coins as dimes and pennies, and encourage them to find the value of the set of coins. They should decide that, since each dime has a value of 10 cents, they can count the value of the dimes by tens; since each penny has a value of 1 cent, they can count the value of the pennies by ones. Have them point to the first dime and read the numeral 10 printed below the coin. Next, they should point to the second dime and read the numeral 20, point to the third dime and read the numeral 30, point to the first penny and read the numeral 31, and point to the second penny and read the numeral 32. Then the children should recount the money, saying, "10, 20, 30, 31, 32 cents." Notice that the word "cents" is used only with the numeral that expresses the total amount. Picture C should be discussed in a similar manner.

For pictures D and E, numerals appear below only the first two coins in each set; for pictures F and G, a numeral appears below only the first coin. Encourage all children to find the value of the sets of coins independently. Then have them count the amount of money aloud. They should point to each coin and name the numeral that should replace each placeholder; for picture D, they should say, "50, 60, 65, 66, 67 cents."

For pictures H and I, neither numerals nor placeholders are given below the coins. However, a placeholder is included for the total amount, and the children should determine the amount of money in a similar manner. Then have them count the amount of money aloud. For picture H, they should say, "50, 60, 65, 70, 75, 76, 77 cents."

Have some real coins available for counting, but do not include sets of coins in which the total value represented is greater than 100 cents. Encourage the children to start counting with the coin that has the greatest value and to use the word "cents" only when expressing the value of the complete set of coins. Have a child arrange a set of coins for counting, and have another child count the amount of money orally. The children should take turns arranging and counting sets of coins.

Practice tablet page 86
Workbook edition page 159

Direct attention to Picture A. Say: "Pretend that the coins in this picture are yours. You are going to count your money to find out how much you have. As you decide what numeral you should say, write this numeral on the response space below the coin. What numeral should you say as you count the two nickels? [10] Be sure to write 10 on the response space below the 2 nickels. What numeral should you say as you count the first nickel? [15] What numeral should you say as you count the next nickel? [20] What should you say as you count the penny? [21].

"For each of the other pictures, pretend the coins are yours and find out how much money you have. On the response space below each coin, write the numeral you should say as you count your money."

Practice tablet page 87
Workbook edition page 160

Say: "For each picture, pretend that the coins are yours and find out how much money you have. On the response space below each coin, write the numeral you should say as you count your money."

ST4 2 page 102
Workbook edition page 161
Finding values represented by sets of coins [extension]

The children continue to find values represented by sets of coins.

On this page, there are five teaching sequences. Each consists of two pictures (A and B; C and D; E and F; G and H; I and J). In the first picture of each sequence, a set of coins is pictured. In the second picture, the coins have been arranged so that the coin with the greatest value is at the left, for easy counting. The numerals are printed below the coins in picture B, but, for the other pictures, the children give the numerals in counting.

For each sequence, have the children study the coins in the first picture and the arrangement of coins in the second picture. Then have them set up a one-to-one correspondence between the coins in the two pictures. They should observe that the same coins appear in each picture. Then have the children find the value of the set of coins. For each sequence, the children should state the numerals used in counting and the value of the set of coins. For picture F, they should say, "25, 35, 45, 50, 55, 56, 57, 58 cents."

Special attention should be given to pictures B, F, and J. The coins could be rearranged so that 1 nickel immediately follows the quarter. This arrangement leads to counting by tens, starting at 30.

Provide many opportunities for the children to find values represented by sets of real coins.

Practice tablet page 88
Workbook edition page 162

Direct attention to the first part of Picture A. Say: "Pretend that these coins are yours. You are to find how much money you have. In counting your money, which coin should you take first? [The quarter] To show that you take the quarter first, write 25 in the first ring. What should you say as you count the quarter? [25] Write 25 on the response space below the ring that shows a quarter. What coin should you take next? [The nickel] How should you show that you take the nickel next? [Write 5 in the second ring.] What should you say as you count the nickel? [30] Write 30 on the response space below the nickel. What coin should you take next? [A penny] How do you show that you take a penny next? [Write 1 in the next ring.] What should you write on the response space below this penny? [31] Complete Picture A. Then do each of the other pictures in the same way."

STA 2 page 103

Finding values represented by sets of coins [extension]

The children continue to find values represented by sets of coins.

Each picture shows a set of coins. Notice that, although the coins are not shown in a horizontal arrangement, their grouping suggests a counting order. For those pictures in which a half dollar is shown, the children should begin counting with 50.

For each picture, first have the children identify the symbolized coins. Then encourage all children to find the value of the set of coins. Next have the children state the numerals used in counting and the value of the set of coins. For picture B, they should say "25, 30, 35, 36, 37, 38, 39 cents."

After all the pictures have been discussed, you may wish to review the ideas of "greater than" and "less than." For example, you might say, "Jim, suppose you have the money shown in picture D. How much money do you have? Bob, suppose you have the money shown in picture I. How much money do you have? Nancy, which boy has more money?"

Provide many opportunities for the children to find values represented by sets of real coins.

Practice tablet page 89
Workbook edition page 163

Say: "Look at Picture A. Pretend that these coins are yours. Count your money. How much money do you have? [38¢] Write 38 on the response space.

"For each of the other pictures, count your money. On the response space, write the numeral that tells how much money you have."

Practice tablet page 90
Workbook edition page 164

Say: "For each picture, pretend that the coins are yours. Count your money. On the response space, write the numeral that tells how much money you have."

STA 2 page 104
Workbook edition page 165

Selection of coins for specified total value [extension]

The children have learned to find the values represented by sets of coins. Now they learn to select from a set of coins those coins whose total value is the same as the price of an item.

Explain that the pictures at the top of the page show sets of coins that children might have and that the pictures at the bottom of the page show items to purchase. Discuss the value of each set of coins and the price of each pictured item.

Direct attention to picture A. Ask the children to identify each coin and to find the total value of the set of coins. Encourage them to imagine that they have these coins and are going to buy one of the items shown in pictures G through L. First, for each item, ask the children if they have enough money to buy the item. Then, for each item that they have enough money to buy, ask what coins they would use to pay for the item. Help them to understand that the coins they select for payment must have the same value as the price of the item. For the coins shown in A, they could buy the picture book (3 dimes and 5 pennies or 3 dimes and 1 nickel) or the musical top (2 dimes, 1 nickel, and 4 pennies), but they could not buy the dog, the purse, the toy horn, or the toy car.

Follow a similar procedure with the set of coins shown in pictures B through E. Notice that, for some sets of coins pictured, there is enough money to buy a toy, but that none of the coins, or combinations of coins, has the same value as the price of the item. For example, the value of the coins shown in picture B is 84 cents. The child has more than enough money to buy the picture book, but he cannot select coins that have the same value as the price of the picture book.

For picture F, the children should discover that they cannot buy any of the items pictured because they cannot select coins that have the same value as the price of any item. Use picture F to introduce the idea of making change.

Practice tablet page 91
Workbook edition page 166

Say: "Look at Picture A. Pretend that you are buying the fish. How much does the fish cost? [48¢] Which coins are you going to use to buy the fish? [4 dimes, 1 nickel, and 3 pennies] Cross out the coins you are going to use.

"For each of the other pictures, pretend you are buying the object shown. Cross out the coins you use to buy the object."

STA 2 page 105
Workbook edition page 167

Making change [extension]

The children learn to make change for amounts less than or equal to a dollar.

It is important that children learn correct procedures in making change. They should learn to select the coins in a systematic manner and to count properly as they select the coins. Be sure that the children begin counting with the price of the article sold or purchased, and that they use coins that bring the first part of the change up to a convenient amount. Convenient amounts are those that are multiples of five (5, 10, 15, and so on).

Two pictures (A and B; C and D; E and F; G and H; I and J) make up each teaching sequence on this page. Direct attention to picture A. First ask the children to tell how much the ball costs. Then have them identify the coin shown. Encourage them to imagine that they have the coin represented and that they want to buy the ball. Ask if they have enough money to pay for the ball, or if they have more money than is needed and should get some back. Explain that the money they get back after buying something is *change*. Do not ask them how much change they should get.

Next direct attention to picture B. Explain that the symbolized coins represent the coins that were given as change for this particular purchase. Get the children to see that in making change they should begin counting with the price of the article and then count the value of the coins until they reach the amount presented in payment. Ask again how much the ball costs. Help the children to understand that, since the ball costs 12 cents, they should say "12" and start counting from this

point. Have them place a finger on the first penny. They should say "13" in counting this penny because, when counting by ones, 13 is the number that is one greater than 12. As they place a finger on the second penny, they should say "14." As they place a finger on the third penny, they should say "15." As they place a finger on the dime, they should say "25 cents." They must continue counting to 25 cents because 25 cents is the amount presented in payment. Point out that in making change it is necessary to say "cents" only as they give the last coin.

It might be well to repeat this activity with real coins or with play money, which the children can make. Let them "trace" real coins on paper and cut out their paper tracings. Their understanding of "change making" will be strengthened if they go through the motions of making change and saying the amount.

Follow a similar procedure for each of the other sequences. Notice that for pictures F, H, and J the children must decide what numerals to use.

Provide classroom activities in which the children have many opportunities to make change.

Practice tablet page 92
Workbook edition page 168

Say: "Pretend you are the clerk in a store. Look at the first part of Picture A. It shows what you have sold. What have you sold? [A bell] It shows you how much the bell costs. How much does the bell cost? [27¢] It shows you how much your customer gave you to pay for the bell. How much did the customer give you? [50¢]

"Now look at the second part of the picture. It shows you the change you should give the customer. What numeral should you say first as you make the change? [27, because the bell cost 27¢] As you decide what numeral you should say, write it on the correct response space. Write 27 on the response space under the bell. What should you say as you count the first penny? [28] What should you say as you count the next penny? [29] What should you say as you count the last penny? [30] What should you say as you count the first dime? [40] What should you say as you count the next dime? [50]

"For each of the other pictures, find out what you have sold, the cost of the object, and how much money the customer gave you. Then write on the correct response space each numeral you should say as you make the change."

STA 2 page 106
Workbook edition page 169
Making change [extension]

The children continue to make change of amounts less than or equal to one dollar. They select the coins they should give in change.

Each of the teaching sequences consists of two pictures (A and B; C and D; E and F; G and H; I and J). The first picture in each sequence shows an item with its price indicated and the money given in payment for the item. The second picture shows the item and a set of coins. From this set of coins, the children select the coins they should use in giving the change. Then they decide what numbers they should name in making change.

Direct attention to picture A. First ask how much the box of crayons costs. Then ask what coin was given to pay for the box of crayons and if the amount it represents is not enough, too much, or if it is the correct amount. Ask if the buyer of the box of crayons will get some change.

Next direct attention to picture B. Have the children identify the pictured coins. Have them select the coins that should be given in change. Ask them to name the number they will say first. Help them to understand that the price of the article is always said

first and that they count the value of the remaining coins until they reach the amount given in payment. They should say "19"; then select a penny and say "20"; then they should select a nickel and say "25 cents." They should understand that in counting the change they will say "19, 20, 25 cents."

Follow a similar procedure for each of the other sequences. Notice that for pictures F, H, and J the children may suggest more than one way to give the correct amount of change.

Have the children use real coins or the play coins they made as they discuss each sequence. Provide classroom activities in which the children have an opportunity to make change.

Practice tablet page 93
Workbook edition page 170

Say: "Again pretend that you are a clerk in a store. Look at the first part of Picture A. What have you sold? [A bag of marbles] What is the cost of the marbles? [37¢] How much money did the customer give you [45¢]

"Now look at the second part of the picture. What is the first numeral you should say as you make the change? [37, because the marbles cost 37¢] As you decide what numeral you should say, write it on the correct response space. Write 37 on the response space below the bag of marbles. What coin should you use first in making the change? [A penny] What numeral should you say as you count the penny? [38] Which coin should you use next? [Another penny] What should you say as you count this penny? [39] Which coin should you use next? [Another penny] What should you say? [40] What coin should you give next? [A nickel] What should you say? [45] Should you give any more coins? [No] Write the symbol for cents after the 45.

"For each of the other pictures, pick out the coins you should give in change. On the correct response space, write the numeral you should say as you make the change. Be sure to write the symbol for cents after the correct numeral. Remember that you do not have to use all the coins shown."

Practice tablet page 94
Workbook edition page 171

Say: "For each picture, pick out the coins you should give in change. On the correct response space, write the numeral you would say as you make the change. Be sure to write the symbol for cents after the correct coin. Remember that you do not have to use all the coins shown."

Practice tablet page 95
Workbook edition page 172

Say: "For each picture, pick out the coins you should give in change. On the correct response space, write the numeral you should say as you make the change. Be sure to write the symbol for cents after the correct numeral. Remember that you do not have to use all the coins shown."

STA 2 page 107
Workbook edition page 173
Equivalent values

The children learn more about the relationship between coins and the values of coins.

Call attention to picture C. Ask: "What coin is represented in picture C? [Each coin represents a nickel.] What is the value of a nickel? [5 cents] How many nickels are there? [20] What is the value of 20 nickels? [100 cents. The children should find the value of the set of coins by counting by fives or by counting by tens and fives.] What is the value of 1 dollar? [100 cents] 1 dollar has the same value as how many nickels? [20] What is the value of 1 dime? [10 cents] 1 dime has

the same value as how many nickels? [2] What is the value of a quarter dollar? [25 cents] 1 quarter dollar has the same value as how many nickels? [5] What is the value of a half dollar? [50 cents] 1 half dollar has the same value as how many nickels? [10]"

The teaching suggestions given for picture C may be adapted for each of the other pictures.

After you have completed the discussion of each picture, direct attention to exercises A through M at the bottom of the page. Introduce the printed word "value." Then have the children read each exercise and decide what numeral replaces the square screen or what word replaces the wavy line. Finally, have the children read the completed sentence aloud.

Practice tablet page 96
Workbook edition page 174

Notice that there are two blocks of exercises on this page. Use your own judgment as to how many exercises you can assign at one time.

Tell the children that they are to complete each exercise by writing on the response space the correct numeral or letter.

STA 2 page 108
Checking up: situations involving additive action and subtractive action -- sentences of the forms 3 + 2 = ■ and 6 - 4 = ■; sums and minuends through 10

The children test their ability to analyze and to solve verbal problems that involve either additive or subtractive action. They also test their knowledge of the addition and subtraction basic facts studied so far.

The problem-solving test on this page includes all the types of problems taught thus far. Only the basic facts studied thus far are included in this block of problems. In this way, computational difficulties are kept to a minimum. Remind the children to think about each problem carefully to "see" what is happening before they write the sentence that describes the problem situation. Results on this achievement test will give you a good overall picture of how well the children understand problem solving and the making of sentences to describe the problem.

Have the children read each problem. Ask them to write the letter that labels it on their papers. Then, for each problem, have them write two sentences. The first sentence should describe the situation and contain a placeholder. The second sentence should contain the numeral that replaces the placeholder. For problem A, they should write the following:

A 7 + 2 = ■ 7 + 2 = 9

For the two blocks of exercises at the right (red letters A through L, and green letters A through M), have the children read each sentence, decide what numeral replaces the square screen, and write the completed sentence on a sheet of paper.

Practice tablet page 97
Workbook edition page 175

Say: "Read Exercise A. Use the picture to find your response. Write your responses on the response spaces. Do each of the other exercises in the same way. For some exercises, you will not use all of the response spaces."

Practice tablet page 98
Workbook edition page 176

Say: "Read each problem. Think about what is happening in the problem. On the first response space, write the sentence that describes the problem. Use a screen to hold the place for the number you are to find. On the second response space, write the sentence with the numeral that belongs where the screen is."

Either before the children begin to work, or after they have completed Problems A through F, direct attention to Exercises A through V. Say: "You are to complete each exercise by writing the correct numeral on the response space."

Practice tablet page 99
Workbook edition page 177

Notice that this page consists of two blocks of exercises. Use your own judgment as to how many of these exercises you can assign at one time.

Tell the children that they are to complete each of the exercises by writing the correct numeral on a response space.

Practice tablet page 100
Workbook edition page 178

Notice that this page consists of two blocks of exercises. Use your own judgment as to how many of these exercises you can assign at one time.

Tell the children that they are to complete each of the exercises by writing the correct numeral on the response space.

1 2 3 4 5 5 7 6 9 10 11 12 13 14 15 15 17 18 19 20 21 22 23 24 25 SH 70 69 68 67 66 65 64

QA 135 H332 1964 RK-2 COMB-TEMP
TCH-GD- PT-1
SEEING THROUGH ARITHMETIC

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CURR
OCT 19 RETURN
OCT 17 RETURN
DEC 2 RETURN
FEB 23 RETURN
NOV 7 RETURN
FEB 25 RETURN
OCT 27 RETURN